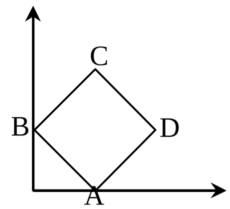
# KVPY QUESTION PAPER-2017 (STREAM SX)

**Date : 19 /11/2017**

## Part A-Mathematics

**1.** Consider a rigid square ABCD as in the figure with A and B on the x and y axis respectively. **[2017]**




When A and B slide along their respective axes, the locus of C forms a part of

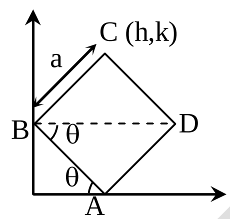
(A) a circle

(B) a parabola

(C) a hyperbola

(D) an ellipse which is not a circle

**Sol.** **[D]**




C(h, k)

$h = a\sin\theta \Rightarrow \sin\theta = \frac{h}{a}$ ...(i)

$k = a\sin\theta + a\cos\theta$

$k = h + a\cos\theta$

$\frac{k-h}{a} = \cos\theta$ ...(2)

$\sin^2\theta + \cos^2\theta = 1$

$\frac{h^2}{a^2} + \frac{(k-h)^2}{a^2} = 1$

$h^2 + (k-h)^2 = a^2$

locus

$x^2 + (y-x)^2 = a^2$

solving we get

$x^2 + y^2 + x^2 - 2xy = a^2$

$2x^2 + y^2 - 2xy = a^2$

check $h^2 - ab < 0$

ellipse

**2.** Among the inequalities below, which ones are true for all natural numbers n greater than 1000? **[2017]**

I. $n! \le n^n$

II. $(n!)^2 \le n^n$

III. $10^n \le n!$

IV. $n^n \le (2n)!$

(A) I and IV only
(B) I, III and IV only
(C) II and IV only
(D) I, II, III and IV

**Sol.** **[B]**

(A) $\frac{n^n}{n!} = \left(\frac{n}{n}\right)\left(\frac{n}{n-1}\right)\left(\frac{n}{n-2}\right)......\left(\frac{n}{1}\right) \ge 1$

$n^n \ge n!$ (correct)

(C) $\frac{n!}{10^n} = \frac{(n)(n-1)(n-2)......}{(10)\,(10)........n \text{ times } (10)}$

given that n > 1000

clearly $\frac{n!}{10^n} \ge 1$

$n! \ge 10^n$

(D)

$\frac{(1.2.3.4........n)(n+1)(n+2)(n+3)......(n+n)}{n^n}$

$(n!)\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\cdots\left(1+\frac{n}{n}\right)$

clearly $\ge 1$.

$2n! \ge n^n$

**3.** Let

$$S = \left\{ \frac{a^2+b^2+c^2}{ab+bc+ca} : a, b, c \in R,\ ab+bc+ca \ne 0 \right\},$$

where R is the set of real numbers. Then S equals **[2017]**

(A) $(-\infty, -1] \cup [1, \infty)$

(B) $(-\infty, 0) \cup (0, \infty)$

(C) $(-\infty, -1] \cup [2, \infty)$

(D) $(-\infty, -2] \cup [1, \infty)$

**Sol.** **[D]**

**Case-I**

$(a-b)^2 + (b-c)^2 + (c-a)^2 \geq 0$

$a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

$a^2 + b^2 + c^2 \geq ab + bc + ca$

If $ab + bc + ca > 0$

Then $\dfrac{a^2 + b^2 + c^2}{ab + bc + ca} \geq 1$

**Case-II**

Let $ab + bc + ca < 0$

$\dfrac{(a+b+c)^2}{ab+bc+ca} \leq 0$

$\dfrac{a^2 + b^2 + c^2 + 2(ab+bc+ca)}{ab+bc+ca} \leq 0$

$\dfrac{a^2 + b^2 + c^2}{ab+bc+ca} + 2 \leq 0$

$\dfrac{a^2 + b^2 + c^2}{ab+bc+ca} \leq -2$

So, Range is $(-\infty, -2] \cup [1, \infty)$

**4.** Let S be the infinite sum given by

$S = \sum_{n=0}^{\infty} \dfrac{a_n}{10^{2n}}$

where $\{a_n\}_{n\geq 0}$ is a sequence defined by $a_0 = a_1 = 1$ and $a_j = 20a_{j-1} - 108\,a_{j-2}$ for $j \geq 2$.

If S is expressed in the form $\dfrac{a}{b}$, where a, b are coprime positive integers, then a equals **[2017]**

(A) 2017 (B) 2020

(C) 2023 (D) 2025

**Sol.** **[D]**

$a_n = 20a_{n-1} - 108a_{n-2}$

$\dfrac{a_n}{10^{2n}} = \dfrac{20a_{n-1}}{10^{2n}} - \dfrac{108a_{n-2}}{10^{2n}}$

$\dfrac{a_n}{10^{2n}} = \dfrac{20}{100}\dfrac{a_{n-1}}{10^{2(n-1)}} - \dfrac{108}{10000}\dfrac{a_{n-2}}{10^{2(n-2)}}$

apply summation

$\sum_{n=2}^{\infty} \dfrac{a_n}{10^{2n}} = \dfrac{1}{5}\sum_{n=2}^{\infty} \dfrac{a_{n-1}}{10^{2(n-1)}} - \dfrac{27}{2500}\sum_{n=2}^{\infty} \dfrac{a_{n-2}}{10^{2(n-2)}}$

$S - 1 - \dfrac{1}{100} = \dfrac{1}{5}(S-1) - \dfrac{27}{2500}S.$

$S - 1 - \dfrac{1}{100} = \dfrac{1}{5}S - \dfrac{1}{5} - \dfrac{27}{2500}S.$

$S\left(1 - \dfrac{1}{5} + \dfrac{27}{2500}\right) = -\dfrac{1}{5} + 1 + \dfrac{1}{100}$

$S\left(\dfrac{2500 - 500 + 27}{2500}\right) = \dfrac{81}{100}$

$S = \dfrac{81 \times 25}{2027}$

$S = \dfrac{2025}{2027}$

**5.** Define a function $f(x) = \dfrac{16x^2 - 96x + 153}{x - 3}$ for all real $x \neq 3$. The least positive value of f(x) is **[2017]**

(A) 16 (B) 18

(C) 22 (D) 24

**Sol.** **[D]**

$y = \dfrac{16x^2 - 96x + 153}{x - 3}$

make it quadratic in x

$16x^2 - x(96 + y) + (153 + 3y) = 0$

$D \geq 0$

Solve $y^2 - 576 \geq 0$

$y \in (-\infty, -24] \cup [24, \infty)$

So, least positive is 24

**6.** Let $n > 2$ be an integer and define a polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1x + a_0$

where $a_0, a_1, \ldots\ldots a_{n-1}$ are integers. Suppose we know that $np(x) = (1 + x)p'(x)$. If $b = p(1)$, then **[2017]**

(A) b is divisible by 10

(B) b is divisible by 3

(C) b is a power of 2

(D) b is a power of 5

**Sol.** **[C]**

$n[x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} - a_1x + a_0]$

$= (1 + x)(nx^{n-1} + a_{n-1}(n-1)x^{n-2} + a_{n-2}x^{n-3}(n-2)$

$+ a_{n-3}(n-3)x^{n-4} + \ldots\ldots)$

compare coefficient of $x^{n-1}$

$na_{n-1} = (n-1)a_{n-1} + n$

Solve $a_{n-1} = n$ or ${}^nC_1$

compare coefficient of $x^{n-2}$

$na_{n-2} = (n-2)a_{n-2} + (n-1)a_{n-1}$

$a_{n-2} = \dfrac{n(n-1)}{2} = {}^nC_2$

similarly $a_{n-3} = {}^nC_3$ & So ......on

$b = P(1) = 1 + a_{n-1} + a_{n-2} + \ldots\ldots a_1 + a_0$

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots {}^nC_n = 2^n$

**7.** The number of 5-tuples (a, b, c, d, e) of positive integers such that

I. a,b,c,d,e are the measures of angles of a convex pentagon in degrees ;

II. $a \le b \le c \le d \le e$ ;

III a, b, c, d, e are in arithmetic progression is **[2017]**

(A) 35 (B) 36 (C) 37 (D) 126

**Sol.** **[B]**

$a + b + c + d + e = 540$

Let say first term = a

Common difference d

$\frac{n}{2}[2a + (n - 1)d] = 540$

$\Rightarrow a + 2d = 108$

**Case-1** $d = 0$

$a = 108$

(108, 108, 108, 108, 108)

**Case-2** $d = 1$

(106, 107, 108, 109, 110)

Similarly it goes up to d = 35

for d > 35, interior angle > 180°

which is not possible

So form d = 0 to d = 35

total 36 tuples are possible

**8.** Thirty two persons $X_1, X_2, \ldots, X_{32}$ are randomly seated around a circular table at equal intervals. Two persons $X_i$ and $X_j$ are said to be within earshot of each other if there are at most three persons between them on the minor arc joining $X_i$ and $X_j$. The probability that $X_1$ and $X_2$ are within earshot of each other is,

$\left(\text{Here} \binom{n}{r} = \frac{n!}{(n-r)!r!}\right)$ **[2017]**

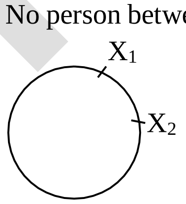
(A) $\frac{\binom{32}{2}30!}{8(32!)}$ (B) $\frac{\binom{32}{2}30!}{4(32!)}$

(C) $\frac{8}{31}$ (D) $\frac{4}{31}$

**Sol.** **[C]**

**Case -1**

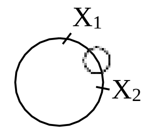
No person between $X_1$ & $X_2$




$\frac{30! \times 2!}{31!} = \frac{2}{31}$

**Case-2**

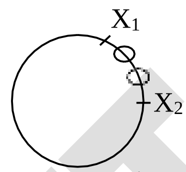
One person between $X_1$ & $X_2$




$\frac{{}^{30}C_1(29)! \times 2!}{31!} = \frac{2}{31}$

**Case-3**

When 2 person between $X_1$ & $X_2$




$\frac{{}^{30}C_2 \times 28! \times 2! \times 2!}{31!} = \frac{2}{31}$

**Case-4**

When 3 person between $X_1$ & $X_2$

$\frac{{}^{30}C_3 \times 27! \times 2! \times 3!}{31!} = \frac{2}{31}$

Total = $\frac{8}{31}$

**9.** Let n be the smallest positive integer such that

$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \ge 4.$

Which one of the following statements is true ? **[2017]**

(A) $20 < n \le 60$ (B) $60 < n \le 80$

(C) $80 < n \le 100$ (D) $100 < n \le 120$

**Sol.** **[A]**

$1 + x < 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$

$1 + x < e^x$

$\ln(1+x) < x$

$x = \frac{1}{y}$

$\ln\left(1 + \frac{1}{y}\right) < \frac{1}{y}$

$\ln(y + 1) - \ln y < \frac{1}{y}$

put y = 1

$\ln(2) - \ln(1) < \frac{1}{1}$

put = y = 2

$\ln 3 - \ln 2 < \frac{1}{2}$

put = y = 3

$\ln 4 - \ln 3 < \frac{1}{3}$

put y = n

$\ln (n+1) - \ln n < \frac{1}{4}$

$\ln (n+1) - \ln (1) < 1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n}$

$\ln (n+1) \leq 4$

$n \leq e^4 - 1$

$n \leq 60$

**10.** A pair of 12-sided fair dice with faces numbered 1,2,3, ....., 12 is rolled. The probability that the sum of the numbers appearing has remainder 2 when divided by 9 is **[2017]**

(A) $\frac{7}{72}$ (B) $\frac{5}{48}$

(C) $\frac{11}{144}$ (D) $\frac{1}{9}$

**Sol.** **[D]**

| $x_1 + x_2 = 11$ or possible cases | $x_1 + x_2 = 20$ |
|---|---|
| (1,10) | (8,12) |
| (2,9) | (9,11) |
| (3,8) | (10,10) |
| (4,7) | |
| (5,6) | |

$\frac{10}{144} + \frac{6}{144} = \frac{16}{144} = \frac{1}{9}$

**11.** Let $x_1, x_2, \ldots\ldots, x_6$ be the roots of the polynomial equation

$x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64 = 0.$

Then **[2017]**

(A) $|x_i| = 2$ for exactly one value of i

(B) $|x_i| = 2$ for exactly two values of i

(C) $|x_i| = 2$ for all values of i

(D) $|x_i| = 2$ for no value of i

**Sol.** **[C]**

It form an G.P.

$$\frac{x^6\left(1 - \left(\frac{2}{x}\right)^7\right)}{\left(1 - \frac{2}{x}\right)} = 0$$

solve that

$x^7 = 2^7$

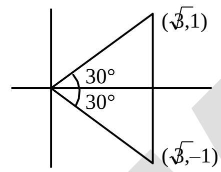
$x = 2$

**12.** In the complex plane, let $z_1 = \sqrt{3} + i$ and $z_2 = \sqrt{3} - i$ be two adjacent vertices of an n-sided regular polygon centered at origin. Then n equals **[2017]**

(A) 4 (B) 6

(C) 8 (D) 12

**Sol.** **[B]**




$\frac{2\pi}{n} = \frac{\pi}{3}$

n = 6

**13.** Let $A^{-1} = \begin{bmatrix} 1 & 2017 & 2 \\ 1 & 2017 & 4 \\ 1 & 2018 & 8 \end{bmatrix}$. Then $|2A| - |2A^{-1}|$ is equal to **[2017]**

(A) 3 (B) –3

(C) 12 (D) –12

**Sol.** **[C]**

$2^3\,|A| - 2^3\,\frac{1}{|A|}$

$|A^{-1}| = \begin{vmatrix} 1 & 2017 & 2 \\ 1 & 2017 & 4 \\ 1 & 2018 & 8 \end{vmatrix}$

$\frac{1}{|A|} = -2 \Rightarrow |A| = \frac{-1}{2}$

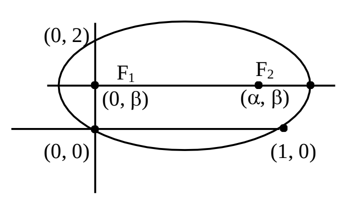
Put the value answer is = 12

**14.** An ellipse with its minor and major axis parallel to the coordinate axes passes through (0,0), (1,0) and (0,2). One of its foci lies on the y-axis. The eccentricity of the ellipse is **[2017]**

(A) $\sqrt{3}$ 1 (B) $\sqrt{5} - 2$

(C) $\sqrt{2}$ 1 (D) $\frac{\sqrt{3}-1}{2}$

**Sol.** **[C]**



Centre is

$$\frac{\left(x-\frac{\alpha}{2}\right)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$$

pass through (0, 0) (0, 2) & (1, 0)

Distance between $F_1$ & $F_2$ $\Rightarrow \alpha = 2ae$

$\alpha^2 = 4a^2 e^2$

Pass through (0, 0)

$$\Rightarrow \frac{\alpha^2}{4a^2} + \frac{\beta^2}{b^2} = 1$$

$$\frac{\alpha^2}{4a^2} = e^2$$

Pass through (0, 2)

$$\frac{\alpha^2}{4a^2} + \frac{(2-\beta)^2}{b^2} = 1$$

from these two $\Rightarrow \beta = 1$

Use properties $PF_1 + PF_2 = 2a$

$F_1(0,1)$ $F_2(\alpha, 1)$ $P(0, 2)$

$1 + \sqrt{\alpha^2+1} = 2a$

$F_1(0,1)$ $F_2(\alpha, 1)$ $P(1, 0)$

$\sqrt{1+1} + \sqrt{(\alpha-1)^2+1} = 2a$

From these two

$\alpha = -2\alpha + 2\sqrt{2}a$

put $\alpha$ in any of above two equations

$a = \frac{\sqrt{2}+1}{2} \Rightarrow 2a = \sqrt{2}+1$

$\alpha = 1$

$\alpha = 2ae$

$\alpha = 1,\ 2a = \sqrt{2}+1$

find $e = \sqrt{2}$

**15.** Let $I_n = \int_0^1 e^{-y} y^n dy$, where n is a non-negative integer. Then $\sum_{n=1}^{\infty} \frac{I_n}{n!}$ is **[2017]**

(A) 1 (B) $1 - \frac{1}{e}$

(C) $\frac{1}{e}$ (D) $1 + \frac{1}{e}$

**Sol.** **[C]**

$I_n = \int_0^1 e^{-y} y^n \, dy$

$I_n = -\frac{1}{e} + n I_{n-1}$ (by reduction formula)

$I_n - n I_{n-1} = -\frac{1}{e}$

$$\frac{I_n}{n!} - \frac{I_{n-1}}{(n-1)!} = -\frac{1}{e\,(n!)}$$

$n = 1$ $\frac{I_1}{1!} - \frac{I_0}{0!} = -\frac{1}{e}$ ...(i)

$n = 2$ $\frac{I_2}{2!} - \frac{I_1}{1!} = -\frac{1}{e(2!)}$ ...(ii)

$n = 3$ $\frac{I_3}{3!} - \frac{I_2}{2!} = -\frac{1}{e(3!)}$ ...(iii)

$n = 4$ $\frac{I_4}{4!} - \frac{I_3}{3!} = -\frac{1}{e(4!)}$ ...(iv)

eq.(1) + eq.(2) × 2 + eq.(3) × 3 + (eq. 4) × 4 + ......

$$-\left\{\frac{I_1}{1!} + \frac{I_2}{2!} + \frac{I_3}{3!} + \frac{I_4}{4!}\right\} - I_0$$

$$= -\frac{1}{e}\left\{1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots\right\}$$

Let $S = \sum_{n=1}^{\infty} \frac{I_n}{n!}$

$-S - I_0 = -\frac{1}{e} \times e$

$S = 1 - I_0$

$S = 1 - \int_0^1 e^{-y} dy$

$S = 1 + \left[e^{-y}\right]_0^1$

$S = 1 + \left[\frac{1}{e} - 1\right]$

$S = \frac{1}{e}$

**16.** The number of solutions of the equation $\sin\theta + \cos\theta = \sin 2\theta$ in the interval $[-\pi, \pi]$ is **[2017]**

(A) 1 (B) 2

(C) 3 (D) 4

**Sol.** **[B]**

Squaring $\sin^2\theta + \cos^2\theta + 2\sin 2\theta = \sin^2 2\theta$

$\Rightarrow \sin^2 2\theta - \sin 2\theta - 1 = 0$

$\sin 2\theta = \left(\frac{1-\sqrt{5}}{2}\right)$, $\sin 2\theta = \frac{\sqrt{5}+1}{2}$ (reject)

two solution exist between

$\frac{-\pi}{2}$ to $\frac{-\pi}{4}$ & $\frac{\pi}{4}$ to 0 (2 solution)

**17.** Let $z_1, z_2, ......, z_7$ be the vertices of a regular heptagon that is inscribed in the unit circle with centre at the origin in the complex plane. Let

$w = \sum_{1 \le i < j \le 7} z_i z_j$, then $|w|$ is equal to **[2017]**

(A) 0 (B) 1

(C) 2 (D) 3

**Sol.** **[A]**

given expression can be written as

$$\frac{(z_1 + z_2 + .... + z_7)^2 - \sum_{i=1}^{7} z_i^2}{2} = \sum_{1 \le i < j \le 7} z_i z_j$$

Sum of all seven root of unity = 0 (By property)

$z_1^2 + z_2^2 + z_3^2 + .... + z_7^2 = 0$ (By property)

**18.** The sound of a cannon firing is heard one second later at a position B than at position A. If the speed of sound is uniform, then **[2017]**

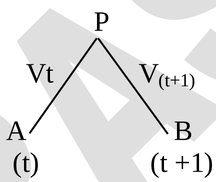
(A) The positions A and B are foci of a hyperbola, with cannon's position on one branch of the hyperbola

(B) the position A and B are foci of an ellipse with cannon's position on the ellipse

(C) One of the positions A,B is focus of a parabola with cannon's position on the parabola

(D) It is not possible to describe the positions of A, B and the cannon with the given information

**Sol. [A]**




PB – PA = Vt + v – vt

PB – PA = V (const)

locus of P is Hyperbola

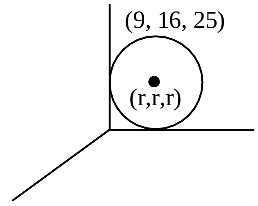
A & B are foci of Hyperbola

**19.** A spherical ball is kept at the corner of a rectangular room such that the ball touches two (Perpendicular) walls and lies on the floor. If a point on the sphere is at distances of 9, 16, 25 from the two walls and the floor, then a possible radius of the sphere is **[2017]**

(A) 13 (B) 15

(C) 26 (D) 36

**Sol.** **[A]**




$(r - 9)^2 + (r - 16)^2 + (r - 25)^2 = r^2$

solving this

$2r^2 - 100\,r + 962 = 0$

$r^2 - 50\,r + 481 = 0$

r = 37, r = 13

possible radius = 13

According to option

**20.** Let m, n be two distinct integers chosen randomly from the set $\{0, 1, 2, ......, 99\}$. Then the probability that $4^m + 4^n + 3$ is divisible by 5 lies in the interval **[2017]**

(A) (0, 0.25] (B) (0.25, 0.5]

(C) (0.5, 0.75] (D) (0.75,1)

**Sol.** **[A]**

possible case

**Case-1**

If unit digit is sum of $4^m + 4^n$ is 7

possible case are

(m, n) = (0, 2) (0, 4)............ (0, 98) = 49

(n, m) = (0,2) (0, 4)................(0,98) = 49

total = 98

**Case-2**

Unit digit is sum of $4^m + 4^n$ is 2

Possible case

(m, n) = (2,4) (2,6)...... (2, 98) = 48

(m, n) = (4, 6) (4, 8)........ (4.98) = 47

⋮

(m, n) = (96, 98) ⇒ **1Case**

same case repetition (n, m)

$2 \times \left(\frac{48 \times 49}{2}\right) = 2352$

Now, Case 1 + Case 2

⇒ 2352 + 98 = 2450

Total number of ways m, n can be selected

= 100 × 99

= 9900

Probability = $\frac{2450}{9900} = 0.2474$

## Section 2-Part A-Physics

**21.** The distance s travelled by a particle in time t is $s = ut - \frac{1}{2}gt^2$ **[2017]**

The initial velocity of the particle was measured to be u = 1 .11 ± 0.01 m/s and the time interval of the experiment was t = 1.01 ± 0.1 s. The acceleration was taken to be g = 9.8 ± 0.1 $m/s^2$ .With these measurements, the student estimates the total distance travelled. How should the student report the result ?

(A) 1.121 ± 0.1 m (B) 1.1 ± 0.1 m

(C) 1.12 ± 0.07 m (D) 1.1 ± 0.07 m

**Sol. [B]**

This question is related to significant numbers as in the question it is asked how student REPORT the result.

On analysis the values of u, t and g the reported result must have three significant number.

Hence correct answer is [B]

**22.** A massive black hole of mass m and radius R is spinning with angular velocity ω. The power P radiated by it as gravitational waves is given by $P = Gc^{-5}m^xR^y\omega^z$, where c and G are speed of light in free space, and the universal gravitational constant , respectively. Then **[2017]**

(A) x = –1, y = 2, z = 4
(B) x = 1, y = 1, z = 4
(C) x = –1, y = 4, z = 4
(D) x = 2, y = 4, z = 6

**Sol. [D]**

$P = ML^2T^{-3}$, $c = LT^{-1}$, $\omega = T^{-1}$, R = L, m = M

$G = M^{-1}L^3T^{-2}$

$[ML^2T^{-3}] = [M^{-1}L^3T^{-2}][LT^{-1}]^{-5}M^xL^yT^{-z}$

solve we get x = 2, y = 4, z = 6

**23.** Consider the following statements for air molecules in an air tight container.

(I) the average speed of molecules is larger than root mean square speed

(II) mean free path of molecules is larger than the mean distance between molecules

(III) mean free path of molecules increases with temperature

(IV) the rms speed of nitrogen molecule is smaller than oxygen molecule

The true statements are : **[2017]**

(A) only II (B) II & III

(C) II & IV (D) I, II & IV

**Sol. [A]**

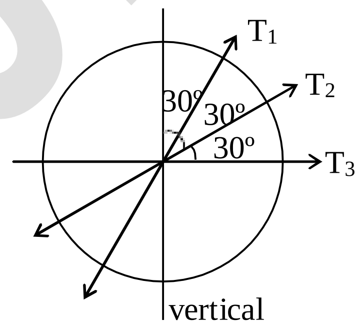
For ideal gas mean free path of molecules is larger than mean distance between molecules

**24.** Three circularly shaped linear polarisers are placed coaxially. The transmission axis of the first polariser is at 30°, the second one is at 60° and the third at 90° to the vertical all in the clockwise sense. Each polariser additionally absorbs 10% of the light. If a vertically polarised beam of light of intensity I = 100 $W/m^2$ is incident on this assembly of polarisers, then the final intensity of the transmitted light will be close to **[2017]**

(A) 10 $W/m^2$ (B) 20 $W/m^2$

(C) 30 $W/m^2$ (D) 50 $W/m^2$

**Sol. [C]**




$I_1 = I_0 \times 0.9\cos^2 30° = I_0 \times 0.9 \times \frac{3}{4}$

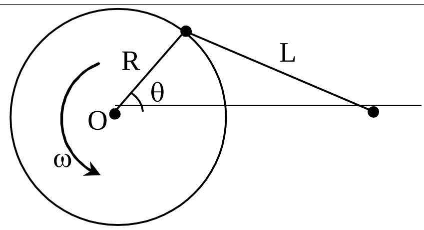
$I_2 = I_1 \times 0.9\cos^2 30° = I_1 \times 0.9 \times \frac{3}{4}$

$I_3 = I_2 \times 0.9\cos^2 30° = I_2 \times 0.9 \times \frac{3}{4}$

$\Rightarrow I_3 = I_0(0.9)^3\left(\frac{3}{4}\right)^3$

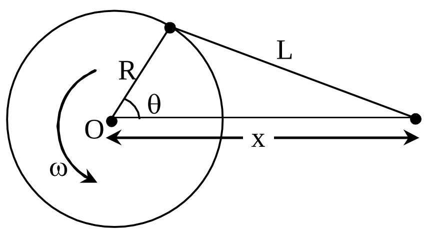
$I_3 = 30.75\ w/m^2$.

**25.** One end of a rod of length L is fixed to a point on the circumference of a wheel of radius R. The other end is sliding freely along a straight channel passing through the centre O of the wheel as shown in the figure below. The wheel is rotating with a constant angular velocity ω about O. Taking $T = \frac{2\pi}{\omega}$ the motion of the rod is **[2017]**




(A) simple harmonic with a period of T
(B) simple harmonic with a period of T/2
(C) not simple harmonic but periodic with a period of T
(D) not simple harmonic but periodic with a period of T/2

**Sol.** **[C]**




$\cos\theta = \dfrac{R^2 + x^2 - L^2}{2Rx}$

$\Rightarrow x^2 = 2Rx \operatorname{cosec}\theta + L^2 - R^2$

displacement of S.H. M. is in the form of

$x = A \sin \omega t + c$

Therefore it is not S. H. M.

It is S. H. M. only which.

It L = R.

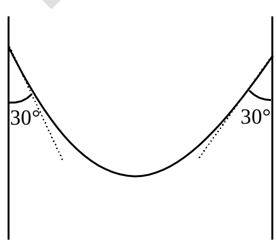
$\Rightarrow x^2 = 2Rx \cos\theta$

$\Rightarrow x = 2R \cos\theta$

$x = 2R \cos \omega t$ [ S.H. M.]

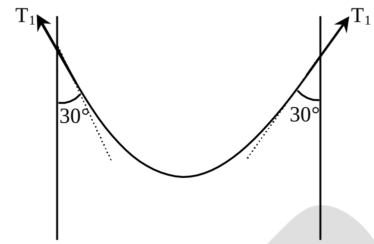
But period of this motion is T

**26.** A rope of mass 5 kg is hanging between two supports as shown. The tension at the lowest point of the rope is close to (take g = 10 m/s$^2$) **[2017]**



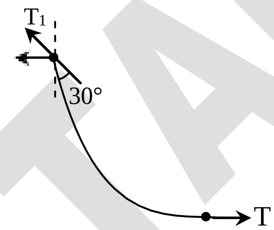

(A) 22 N (B) 44 N
(C) 28 N (D) 14 N

**Sol.** **[D]**




$\therefore\ 2T_1 \cos 30° = mg = 5 \times 10 = 50.$

$2T_1 \cos 30° = 50$

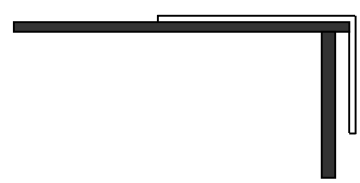



$T = T_1 \sin 30°$

$= \dfrac{50}{2\sqrt{3}/2} \cdot \dfrac{1}{2} = \dfrac{25}{\sqrt{3}} = \dfrac{25\sqrt{3}}{3}$

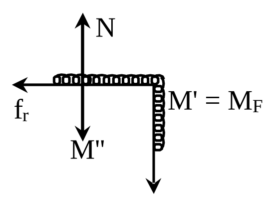
T = 14.41 N

**27.** A uniform rope of total length *l* is at rest on a table with fraction f of its length hanging (see figure). If the coefficient of friction between the table and the chain is μ then **[2017]**



(A) $f = \mu$ (B) $f = 1/(1 + \mu)$
(C) $f = 1/(1 + 1/\mu)$ (D) $f = 1/(\mu + 1/\mu)$

**Sol.** **[C]**




$f_r = \mu N = \mu M'' g = \mu (M - M_f) g$

at equilibrium.

$f_r = M'g$

$\mu M(1 - f)g = M_F g$

$\mu(1 - f) = f$

$\mu = (\mu + 1) f$

$f = \dfrac{1}{\left(1 + \dfrac{1}{\mu}\right)}$

**28.** A light beam travelling along the x axis with planar wavefront is incident on a medium of thickness t. In the region, where light is falling the refractive index can be taken to be varying such that $\frac{dn}{dy} > 0$. The light beam on the other side of the medium will emerge **[2017]**

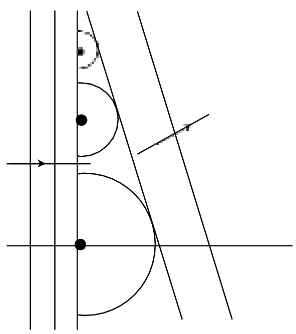
(A) parallel to the x-axis

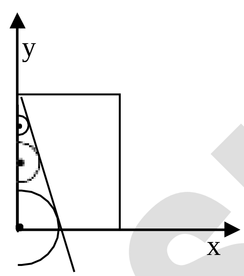
(B) bending downward

(C) bending upward

(D) split into two or more beams

**Sol.** **[C]**






In y direction refractive index is increasing therefore speed of light is decreasing

**29.** Let the electrostatic field E at distance r from a point charge q not be an inverse square but, instead an inverse cubic, e.g. $E = k\frac{q}{r^3}\hat{r}$

Here k is a constant. Consider the following two statements **[2017]**

(i) Flux through a spherical surface enclosing the charge is $\phi = q_{enclosed}/\in_0$

(ii) A charge placed inside uniformly charged shell will experience a force.

Choose the correct option.

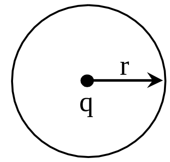
(A) Only (i) is valid

(B) Only (ii) is valid

(C) Both (i) and (ii) are invalid

(D) Both (i) and (ii) are valid

**Sol.** **[B]**

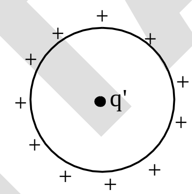



$|E| = \frac{kq}{r^3}$

$\therefore d\phi = E \,.\, \overrightarrow{dS}$

$\phi = \int d\phi = \int \vec{E}.\overrightarrow{dS} = \frac{kq}{r^3} \,.\, 4\pi r^2$

$\phi = \frac{kq4\pi}{r} \neq \frac{q_{en}}{\varepsilon_0}$




force on q' is zero

**30.** A star of mass M and radius R is made up of gases. The average gravitational pressure compressing the star due to gravitational pull of the gases making up the star depends on R as **[2017]**

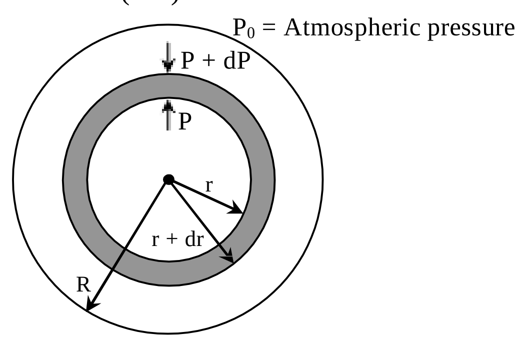
(A) $\frac{1}{R^4}$ (B) $\frac{1}{R}$

(C) $\frac{1}{R^2}$ (D) $\frac{1}{R^6}$

**Sol.** **[A]**

Consider a spherical shell of radius r and radial thickness dr. P & P + dP are pressure at its inner and outer surface.

Let $g_r$ = gravitational acceleration at distance r (< R)



For equilibrium of this shell -

$(P + dP)(4\pi r^2) + (4\pi r^2\, dr)\,\rho\, g_r = (P)(4\pi r^2)$

$$\left\{\rho = \frac{3M}{4\pi R^3} = \text{Density of sphere}\right\}$$

$\Rightarrow\ dP = -\rho\, g_r\, dr$

$\because\ \ g_r = \frac{4}{3}\pi G\rho r$

$\Rightarrow dP = -\frac{4}{3}\pi G\rho^2 r\, dr$

{(–) ve sign indicates that pressure is decreasing with radius}

$$\Rightarrow \int_{P}^{P_0} dP = \int_{r}^{R} -\frac{4}{3}\pi G\rho^2\, r\, dr$$

$$\Rightarrow P_0 - P = -\frac{4}{3}\pi G\rho^2\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

$$\Rightarrow\ P = P_0 + \frac{4}{3}\pi G\rho^2\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

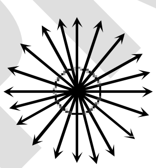
$$P = P_0 + \frac{3GM^2}{8\pi R^4}\left(1 - \frac{r^2}{R^2}\right)$$

Hence $P \propto \frac{1}{R^4}$

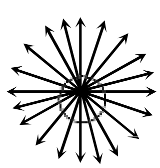
$\Rightarrow$ Average pressure will also be proportional to $\frac{1}{R^4}$

$\Rightarrow$ correct answer is [A]

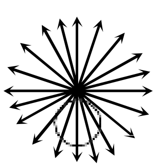
**31.** The black shapes in the figure below are closed surfaces. The electric field lines are in red. For which case the net flux through the surfaces is non-zero ? **[2017]**



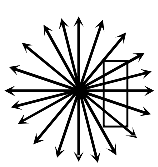
(a)



(b)



(c)



(d)

(A) In all cases net flux is non-zero

(B) Only (c) and (d)

(C) Only (a) and (b)

(D) Only (b), (c) and (d)

**Sol.** **[C]**

$\phi = \frac{q_{en}}{\varepsilon_0}$

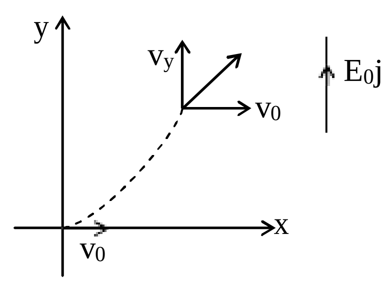
$q_{en}$ is not zero for a and b. therefore flux ($\phi$) is not zero.

**32.** A particle of charge q and mass m enters a region of a transverse electric field of $E_0\hat{j}$ with initial velocity $v_0\hat{i}$. The time taken for the change in the de Broglie wavelength of the charge from the initial value of $\lambda_0$ to $\lambda_0/3$ is proportional to **[2017]**

(A) $\frac{q}{m}$  (B) $\frac{m}{q}$

(C) $\sqrt{\frac{q}{m}}$  (D) $\sqrt{\frac{m}{q}}$

**Sol.** **[B]**




$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{v}$

$v = \sqrt{v_y^2 + v_0^2}$

$v_y = u_y + a_y t$

$v_y = 0 + \frac{qE_0}{m}t$

$(3v_0) = \sqrt{v_y^2 + v_0^2}$

$\Rightarrow v_y^2 = 8v_0^2$

$\Rightarrow \frac{qE_0}{m}t = 2\sqrt{2}\, v_0$

$\Rightarrow t = \frac{2\sqrt{2}m}{qE_0}v_0$

$t \propto \frac{m}{q}$

**33.** Consider the following nuclear reactions :

**I.** $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + X$

**II.** $^{9}_{4}Be + ^{4}_{2}H \rightarrow ^{12}_{6}He + Y$

Then **[2017]**

(A) X and Y are both protons.

(B) X and Y are both neutrons.

(C) X is a proton and Y is a neutron.

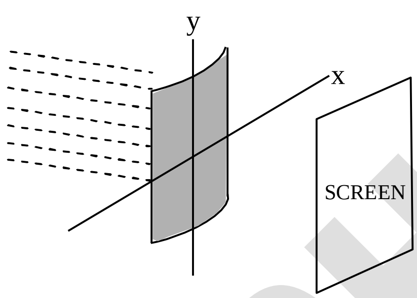
(D) X is a neutron and Y is a proton.

**Sol.** **[C]**

$_7N^{14} + {_2He^4} \rightarrow {_8O^{17}} + {_1H^1}$

$_4Be^9 + {_2H^4} \rightarrow {_6He^{12}} + {_0n^1}$

**34.** Consider a plane parallel beam of light incident on a plano-cylindrical lens as shown below. Which of the following will you observe on a screen placed at the focal plane of the lens ? **[2017]**



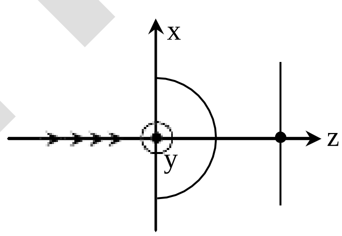

(A) The screen will be uniformly illuminated.

(B) There will be a single bright spot on the screen.

(C) There will be a single bright line on the screen parallel to the x-axis

(D) There will be a single bright line on the screen parallel to the y-axis

**Sol.** **[D]**




no deflection of light beam

**35.** The n-side of the depletion layer of a p-n junction : **[2017]**

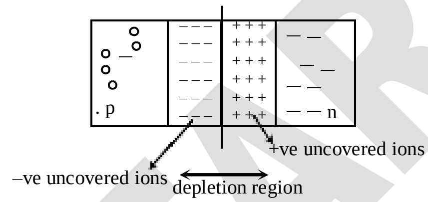
(A) always has same width as of the p-side.

(B) has no bound charges.

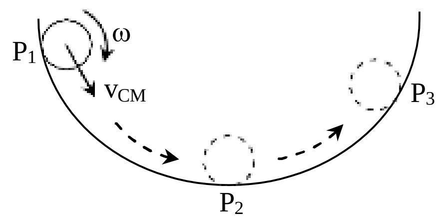
(C) is negatively charged.

(D) is positively charged.

**Sol.** **[D]**




**36.** A small ring is rolling without slipping on the circumference of a large bowl as shown in the figure. The ring is moving down at $P_1$, comes down to the lower most point $P_2$ and is climbing up at $P_3$. Let $\nu_{CM}$ denote the velocity of the centre of mass of the ring. Choose the correct statement regarding the frictional force on the ring. **[2017]**




(A) It is opposite to $\nu_{CM}$ at the points $P_1$, $P_2$ and $P_3$.

(B) It is opposite to $\nu_{CM}$ at $P_1$ and in the same direction as $\nu_{CM}$ at $P_3$.

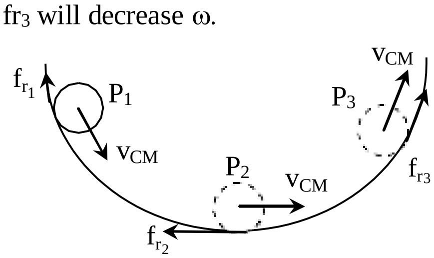
(C) It is in the same direction as $\nu_{CM}$ at $P_1$ and opposite to $\nu_{CM}$ at $P_3$.

(D) It is zero at the points $P_1$, $P_2$ and $P_3$.

**Sol.** **[B]**

$fr_1$ will increase ω.

$fr_3$ will decrease ω.



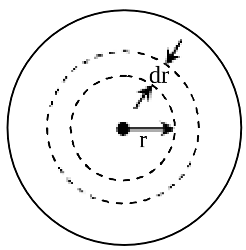
**37.** A bomb explodes at time t = 0 in a uniform, isotropic medium of density ρ and releases energy E, generating a spherical blast wave. The radius R of this blast wave varies with time t as : **[2017]**

(A) t (B) $t^{2/5}$
(C) $t^{1/4}$ (D) $t^{3/2}$

**Sol. [B]**

The energy will propagate in form of spherical blast wave which is longitudinal in nature.

Hence velocity of propagate of disturbance




$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Where P = Pressure

$\because PV = nRT$

$$v = \sqrt{\frac{\gamma RT}{V\rho}}$$

Where $V = \frac{4\pi r^3}{3}$

& v = velocity of propagation = $\frac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt} = \sqrt{\frac{\gamma RT}{\left(\frac{4}{3}\pi r^3\right)\rho}}$$

$$\Rightarrow \frac{dr}{dt} = \frac{k}{r^{3/2}}$$

k = constant = $\sqrt{\frac{3\gamma RT}{4\pi\rho}}$

$\Rightarrow r^{3/2}dr = kdt$

$$\int r^{3/2}dr = \int kdt$$

$r^{5/2} = kt \Rightarrow r = (kt)^{2/5}$

$\Rightarrow r \propto t^{2/5}$

Hence correct answer is (B)

**Alternate solution**

$R \propto E^a P^b t^c$

Dimension of R = L

$L \propto (ML^2 T^{-2})^a (ML^{-3})^b (T)^c$

$2a - 3b = 1$ ...(i)

$c - 2a = 0$ ...(ii)

$a + b = 0$ ...(iii)

from (i), (ii) & (iii)

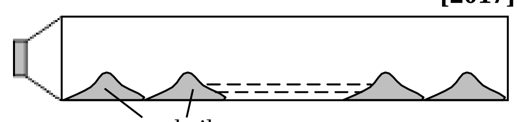
$a = \frac{1}{5}$

$c = 2a$

$\therefore c = \frac{2}{5}$

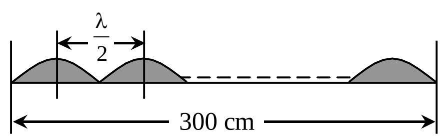
$R \propto t^{2/5}$

**38.** A closed pipe of length 300 cm contains some sand. A speaker is connected at one of its ends. The frequency of the speaker at which the sand will arrange itself in 20 equidistant piles is close to (velocity of sound is 300 m/s) **[2017]**




(A) 10 kHz (B) 5 kHz
(C) 1 kHz (D) 100 kHz

**Sol. [C]**




$20\frac{\lambda}{2} = 300$

λ = 30 cm = 0.3 cm

v = νλ

$v = \frac{300}{0.3} = 1000$ Hz

v = 1kHz

**39.** A planet of radius $R_p$ is revolving around a star of radius $R^*$, which is at temperature $T^*$. The distance between the star and the planet is d. If the planet's temperature is $f\ T^*$, then $f$ is proportional to **[2017]**

(A) $\sqrt{R^*/d}$ (B) $R^*/d$
(C) $R^* R_p/d^2$ (D) $(R^*/d)^4$

**Sol.** **[A]**

The planet should be in Thermal equilibrium with star.

Amount of heat energy emitted by start per second $E_1 = \left(\sigma T^{*4}\right)\left(4\pi R^{*2}\right)$

$\because$ d = distance between star and planet

Hence, amount of energy reaching planet per unit area per second $= \dfrac{E_1}{4\pi d^2}$

$$= \frac{\left(\sigma T^{*4}\right)\left(4\pi R^{*2}\right)}{4\pi d^2}$$

$$= \frac{\sigma T^{*4} R^{*2}}{d^2}$$

Hence energy received by planet per second

$$E_2 = \left(\frac{\sigma T^{*4} R^{*2}}{d^2}\right)\pi R_P^2$$

T = Temperature of planet then amount of energy emitted by planet per second

$$E_3 = \left(\sigma T^4\right)\left(4\pi R_P^2\right)$$

For thermal equilibrium $E_3 = E_2$

$$\Rightarrow \left(\sigma T^4\right)\left(4\pi R_P^2\right) = \left(\frac{\sigma T^{*4} R^{*2}}{d^2}\right)\left(\pi R_p^2\right)$$

$$\Rightarrow T^4 = \frac{T^{*4} R^{*2}}{4d^2}$$

$$\Rightarrow T = T^{*}\sqrt{\frac{R^{*}}{2d}} = fT^{*}$$

$$\Rightarrow f \propto \sqrt{\frac{R^{*}}{d}}$$

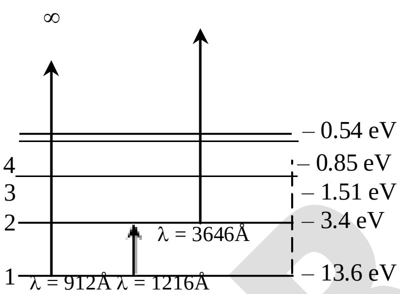
Hence correct answer is (A)

**40.** Some of the wavelength observed in the emission spectrum of neutral hydrogen gas are 912, 1026, 1216, 3646, 6563 Å. If broad band light is passing through neutral hydrogen gas at room temperature, the wavelength that will not be absorbed strongly is **[2017]**

(A) 1026 Å (B) 1216 Å

(C) 912 Å (D) 3646 Å

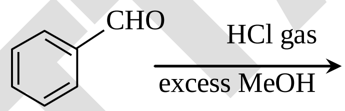
**Sol.** **[D]**

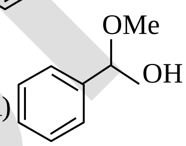


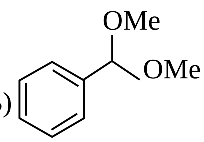

Since hydrogen is in it's neutral state. Therefore $\lambda$ = 3646 Å will not strongly absorbed

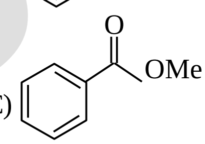
## Section 3-Part A Chemistry

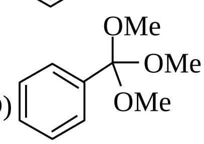
**41.** The major product formed in the following reaction is **[2017]**




(A) 

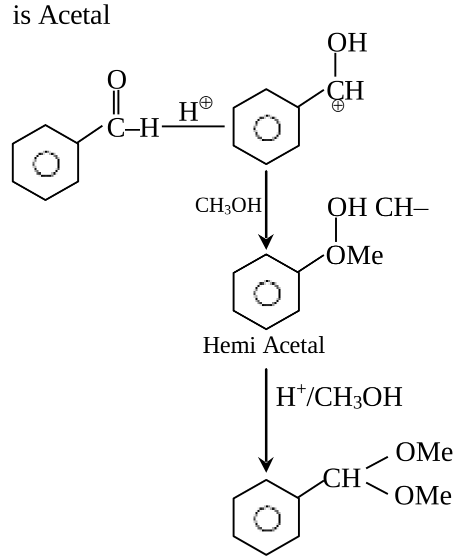

(B) 


(C) 


(D) 


**Sol.** **[B]**

It is example of Nucleophilic addition in which Alcohol attack as Nueleophile and final product is Acetal **[2017]**

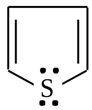



**42.** Which among the following is a non-benzenoid aromatic compound ? **[2017]**

(A) o-Xylene (B) Phenanthrene

(C) Indole (D) Thiophene

**Sol.** **[D]**

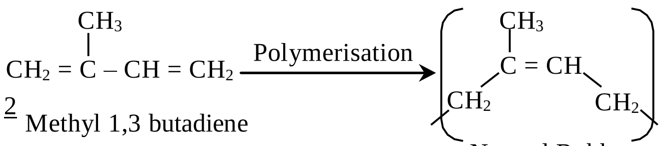


It is Non-Benzenoid compound

**43.** Natural rubber is a polymer of **[2017]**

(A) Neoprene (B) Chloroprene
(C) Isoprene (D) Styrene

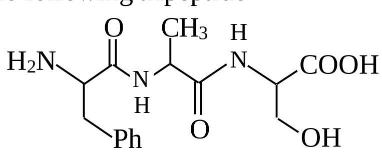
**Sol.** **[C]**




2 Methyl 1,3 butadiene
[Isoprene]

Natural Rubber
Poly isoprene
( cis form)

**44.** The following tripeptide




can be represented as **[2017]**

(A) Tyr-Val-Thr (B) Phe-Ala-Ser
(C) Phe-Leu-Cys (D) Lys-Ala-Ser

**Sol.** **[B]**

Phenyl alanine $NH_2 - \underset{|}{\overset{COOH}{CH}} - CH_2 - Ph$

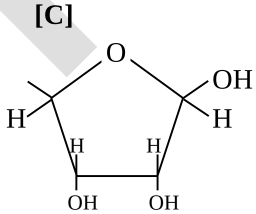
Alanine $NH_2 - \overset{CH_3}{CH} - COOH$

Serine $NH_2 - \underset{CH_2-OH}{CH} - COOH$

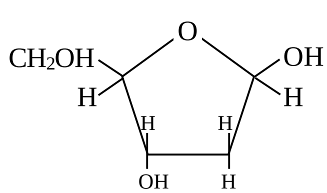
**45.** The sugar units present in natural DNA and RNA, respectively, are **[2017]**

(A) D-2-deoxyribose and L-ribose
(B) L-2-deoxyribose and D-ribose
(C) D-2-deoxyribose and D-ribose
(D) L-2-deoxyribose and L-ribose

**Sol.** **[C]**




β-D Ribose
(in RNA)




β-2 Deoxy Ribose

**46.** The major product formed in the following reaction is **[2017]**

$CH_3Br + CH_3CH_2ONa \rightarrow$

(A) $CH_3CH_2CH_2OH$
(B) $CH_3OCH_3$
(C) $CH_3CH_2OCH_3$
(D) $CH_3CH_2OCH_2Br$

**Sol.** **[C]**

It is Williamson synthesis Reaction ($SN^2$ $rex^n$)

$CH_3 - Br + CH_3 - CH_2 - \overset{\ominus}{O}\,Na^{\oplus}$

$\downarrow \ominus$

$CH_3{-}CH_2 — O......\overset{\delta+}{CH_3}........\overset{\delta-}{Br}$ Transition state

$\downarrow$

$CH_3{-}CH_2 - O - CH_3$
Ether

**47.** The most abundant metal ion present in the human body is **[2017]**

(A) $Zn^{2+}$ (B) $Ca^{2+}$
(C) $Na^{+}$ (D) $Fe^{2+}$

**Sol.** **[B]**

Calcium Mainly present in Bones all other are required in lower amount

**48.** Phosphorous reacts with chlorine gas to give a colourless liquid, which fumes in moist air to produce HCl and **[2017]**

(A) $POCl_3$ (B) $H_3PO_3$
(C) $PH_3$ (D) $H_3PO_4$

**Sol.** **[B]**

$P_4 + Cl_2 \longrightarrow PCl_3$
colourless liquid

$\downarrow$ $H_2O$ moisture

$H_3PO_3 + HCl$

**49.** The oxidising ability of the given anions follows the order **[2017]**

(A) $TiO_4^{4-} < VO_4^{3-} < CrO_4^{2-} < MnO_4^{-}$
(B) $VO_4^{3-} < CrO_4^{2-} < MnO_4^{-} < TiO_4^{4-}$
(C) $CrO_4^{2-} < MnO_4^{-} < VO_4^{3-} < TiO_4^{4-}$
(D) $VO_4^{3-} < TiO_4^{4-} < CrO_4^{2-} < MnO_4^{-}$

**Sol.** **[A]**

It is decided by SRP value

**50.** The complete hydrolysis of $XeF_6$ results in the formation of **[2017]**

(A) $XeO_2F_2$ (B) $XeOF_4$
(C) $XeO_3$ (D) $XeO_2$

**Sol.** **[C]**

$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

white

explosive compound

**51.** The reactivity of the following compounds towards water is in the order **[2017]**

(A) $Cl_2O_7 < P_2O_5 < B_2O_3$

(B) $B_2O_3 < P_2O_5 < Cl_2O_7$

(C) $P_2O_5 < B_2O_3 < Cl_2O_7$

(D) $B_2O_3 < Cl_2O_7 < P_2O_5$

**Sol. [B]**

These dissolve in water to form hydroxy acid. Stronger acidic oxide react more faster Acidic strength increase with increase in EN

**52.** Among the following complexes, the one that can exist as facial (*fac*) and meridional (*mer*) isomers is **[2017]**

(A) $[Co(NO_2)_3(NH_3)_3]$

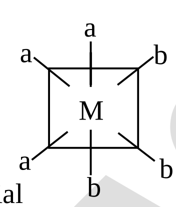
(B) $K_3[Fe(CN)_6]$

(C) $[Co(H_2O)_2(NH_3)_4]Cl_3$

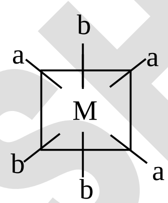
(D) $[CoCl(NH_3)_5]Cl_2$

**Sol.** **[A]**

$Ma_3b_3$ Type exist in facial and meridional (mer)




Facial




Meridional

Example $[Co\ (NO_2)_3\ (NH_3)_3]$

**53.** An excess of $Ag_2CrO_4(s)$ is added to a $5 \times 10^{-3}$ M $K_2CrO_4$ solution. The concentration of $Ag^+$ in the solution is closest to

[Solubility product for $Ag_2CrO_4 = 1.1 \times 10^{-12}$]

**[2017]**

(A) $2.2 \times 10^{-10}$ M

(B) $1.5 \times 10^{-5}$ M

(C) $1.0 \times 10^{-6}$ M

(D) $5.0 \times 10^{-3}$ M

**Sol.** **[B]**

$1.1 \times 10^{-12} = [Ag^+]^2\ [5 \times 10^{-13}]$

$\therefore [Ag^+] = 1.5 \times 10^{-5}$

**54.** The packing efficiency in a body-centred cubic (bcc) structure is closet to **[2017]**

(A) 74 % (B) 63%

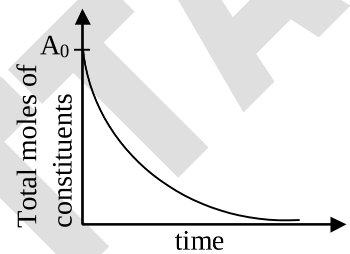
(C) 68 % (D) 52%

**Sol. [C]**

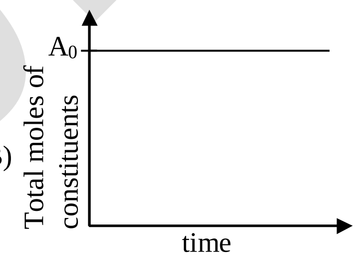
$$\eta = \frac{2 \times 4/3\pi R^3}{\left(\frac{4R}{\sqrt{3}}\right)^3} \times 100 \cong 68\%$$

**55.** The consecutive reaction $X \rightarrow Y \rightarrow Z$ takes place in a closed container. Initially, the container has $A_0$ moles of X (and no Y and Z). The plot of total moles of the constituents in the container as a function of time will be **[2017]**

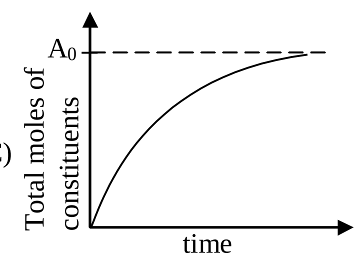
(A)



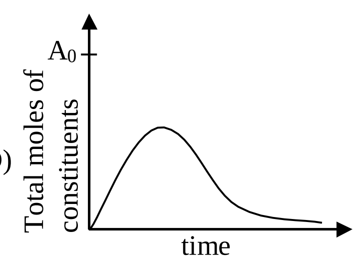

(B)




(C)




(D)




**Sol.** **[B]**

| | X | $\longrightarrow$ Y | $\longrightarrow$ Z |
|---|---|---|---|
| t = 0 | $A_0$ | 0 | 0 |
| t = t | $A_0 - x$ | y | z |

So total moles of constituents will be more at any time 't' as compared to t = 0

**56.** The particle emitted during the sequential radioactive decay of $^{238}U_{92}$ to $^{206}Pb_{82}$ are **[2017]**

(A) 5 $\alpha$ and 6 $\beta$ (B) 6 $\alpha$ and 8 $\beta$

(C) 8 $\alpha$ and 4 $\beta$ (D) 8 $\alpha$ and 6 $\beta$

**Sol.** **[D]**

no of $\alpha$ particle = $\dfrac{238 - 206}{4} = 8$

no of $\beta$-particle = 6

**57.** The allowed set of quantum numbers for an electron in a hydrogen atom is **[2017]**

(A) n = 4, $l$ = 2, $m_l$ = 0, $m_s$ = 0

(B) n = 3, $l$ = 1, $m_l$ = –3, $m_s$ = –1/2

(C) n = 3, $l$ = 3, $m_l$ = –1, $m_s$ = 1/2

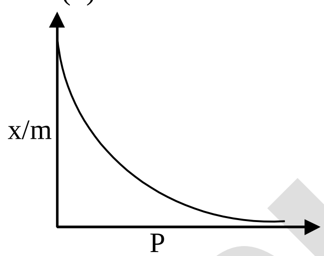
(D) n = 2, $l$ = 1, $m_l$ = –1, $m_s$ = 1/2

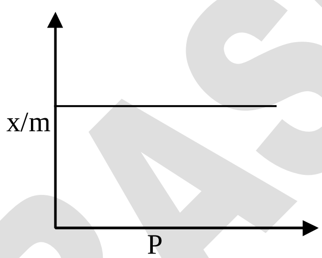
**Sol.** **[D]**

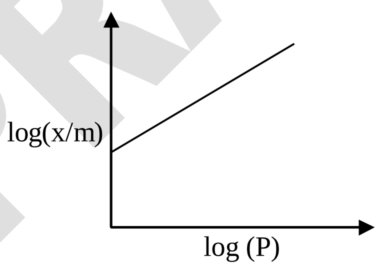
n = 2 $\ell$ = 0 m = 0 $m_s = \pm 1/2$

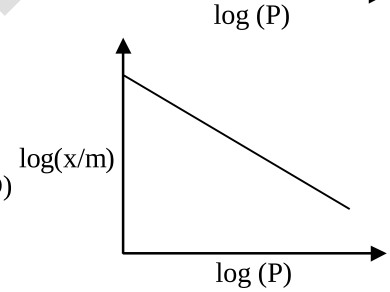
1 m = –1, 0, + 1 $m_s = \pm 1/2$

**58.** The plot that best represents the relationship between the extent of adsorption (x/m) and pressure (P) is **[2017]**

(A)



(B)



(C)



(D)



**Sol.** **[C]**

$\dfrac{x}{m} \propto P^{1/n}$

$\dfrac{x}{m} = k\,P^{1/n}$

$\log \dfrac{x}{m} = \log k + \dfrac{1}{n} \log P$

y = c + mx

**59.** The pH of 0.1 M acetic acid solution is closest to [Dissociation constant of acid $K_a = 1.8 \times 10^{-5}$] **[2017]**

(A) 2.87 (B) 1.00

(C) 2.07 (D) 4.76

**Sol.** **[A]**

$[H^+] = \sqrt{K_a.c} = \sqrt{1.8 \times 10^{-5} \times 0.1} = \sqrt{1.8} \times 10^{-3}$

pH = 3–log (1.34) = 2.87

**60.** The limiting molar conductivities of the given electrolytes at 298 K follow the order

$[\lambda^0 (K^+) = 73.5, \lambda^0 (Cl^-) = 76.3,$

$\lambda^0 (Ca^{2+}) = 119.0, \lambda^0 (SO_4^{2-}) = 160.0$ S cm$^2$ mol$^{-1}$] **[2017]**

(A) $KCl < CaCl_2 < K_2SO_4$

(B) $KCl < K_2SO_4 < CaCl_2$

(C) $K_2SO_4 < CaCl_2 < KCl$

(D) $CaCl_2 < K_2SO_4 < KCl$

**Sol.** **[A]**

$\lambda^\circ_{KCl} = 73.5 + 76.3 = 149.8$

$\lambda^\circ_{CaCl_2} = 119 + 2 \times 76.3 = 271.6$

$\lambda^\circ_{K_2SO_4} = 2 \times 73.5 + 160 = 307$

**Section 4-PartA-Biology**

**61.** Resting membrane potential of a neuron is approximately **[2017]**

(A) – 70 mV (B) + 70 mV

(C) – 0.7 V (D) + 0.7 V

**Sol. [A]**

Resting membrane potential is potential deference across the plasma membrane when neuron is at rest.

**62.** Amphimixis is **[2017]**

(A) A fusion of pronuclei of male gametes.

(B) a fusion of pronuclei from male and female gametes

(C) a fusion of pronuclei of female gametes

(D) the development of a somatic cell into an embryo

**Sol.** **[B]**

Amphimixis → Formation of offspring due to fusion of ♂ & ♀ gametes

**63.** Activation of sympathetic nervous system **[2017]**

(A) decreases blood pressure.

(B) causes pupil contraction.

(C) increases heart rate.

(D) causes bronchoconstriction.

**Sol.** **[C]**

Sympathetic nervous system stimulate Sino-Atrial Node.

**64.** At physiological temperature, sterols in biological membranes **[2017]**

(A) increase their fluidity.

(B) decrease their fluidity.

(C) increase their permeability to water.

(D) decrease their permeability to water.

**Sol.** **[A]**

Cholesterol in eukaryote & hapanoid in prokaryote decreases membranes fluidity

**65.** Which ONE of the following is a hetero-polysaccharide ? **[2017]**

(A) Glycogen (B) Starch

(C) Cellulose (D) Hyaluronic acid

**Sol. [D]**

Hyaluronic acid is polymer of D-Guluconic acid, N-Acetyl, D-Glucose amine. So heteropolysaccharide.

**66.** Bacterial plasmids are genetic entities that, **[2017]**

(A) are non-transferable to the same bacterial species.

(B) are capable of independent replication.

(C) have RNA as genetic material.

(D) always require integration in the genome for their replication.

**Sol.** **[B]**

It is extra chromosomal genetic material capable of independent replication.

**67.** Skin-prick test on the forearm is conducted to identify the responsible allergen. This is because **[2017]**

(A) of the presence of mast cells under the skin.

(B) lymphocytes migrate rapidly from the blood to the skin.

(C) hair follicles can enhance the reaction.

(D) Neutrophils migrate rapidly from the blood to the skin.

**Sol.** **[A]**

Mast Cells release histamine.

**68** Which ONE of the following processes in *E coli* does NOT directly involve RNA ? **[2017]**

(A) DNA replication (B) Transcription

(C) Translation (D) DNA repair

**Sol.** **[D]**

RNA primers are involved in DNA replication

**69.** Which ONE of the following statements is INCORRECT for translation in cytoplasm ? **[2017]**

(A) One codon codes for only one amino acid.

(B) One amino acid may be coded by many codons.

(C) More than one amino acids are coded by one specific condon.

(D) There are some codons that do not code for any amino acid.

**Sol.** **[C]**

One specific codon codes for only one aminoacid.

**70.** Two homozygous parents harboring two different alleles of a gene, exhibiting incomplete dominance for flower colour were used for a genetic experiment. Which ONE of the following statements is INCORRECT ? **[2017]**

(A) The $F_2$ generation will consist of plants of three different flower colours

(B) The genotypic and phenotypic ratios obtained in the $F_2$ generation will be different

(C) The $F_1$ generation will be of a different flower colour compared to both the parents

(D) The genotypic ratio obtained in the $F_2$ generation will be the same irrespective of whether it is complete dominance or incomplete dominance

**Sol. [B]**

Both genotype & phenotype ratio same 1 : 2 : 1

**71.** Which ONE of the following is an essential condition for a population to be at Hardy-Weinberg equilibrium ? **[2017]**

(A) Random mating
(B) Immigration
(C) Emigration
(D) Geographical isolation

**Sol.** **[A]**
For Random mixing of alleles.

**72.** Inbreeding in a population leads to **[2017]**

(A) decrease in recessive disorders
(B) heterosis
(C) increase in homozygosity
(D) increase in heterozygosity

**Sol.** **[C]**
Inbreeding is the production of offspring from the mating or breeding of individuals or organisms that are closely related genetically

**73.** Which ONE of the following molecules serves as a substrate for direct synthesis of ATP ? **[2017]**

(A) 1, 3-bisphosphoglycerate
(B) Glucose 6-phosphate
(C) Pyruvate
(D) Fructose 1,6-bisphosphate

**Sol.** **[A]**

$$\text{1,3-biPGA} \xrightarrow[\text{ADP} \rightarrow \text{ATP}]{} \text{3 PGA}$$

**74.** If a pure chlorophyll solution is illuminated with ultraviolet light, the solution appears **[2017]**

(A) green (B) violet
(C) red (D) black

**Sol.** **[C]**
Fact based answer

**75.** Botanical names of plants are given in Column-I, and the family/order name in Column-II. Choose the appropriate combination from the options below **[2017]**

| Column-I | Column-II |
|---|---|
| (P) *Tamarindus indica* | (i) Arecaceae |
| (Q) *Cocos nucifera* | (ii) Liliaceae |
| (R) *Colchicum automnale* | (iii) Solanaceae |
| (S) *Withania somnifera* | (iv) Papilionaceae |

(A) P-iv, Q-i, R-ii, S-iii
(B) P-iv, Q-ii, R-iii, S-i
(C) P-i, Q-ii, R-iv, S-iii
(D) P-iv, Q-i, R-iii, S-ii

**Sol.** **[A]**
Fact based answer

**76.** Nitrogen fixation is inhibited by oxygen. However, in aerobic nitrogen fixing bacteria, nitrogen is fixed in the presence of oxygen. Nitrogenase in such organisms is protected by which ONE o the following mechanisms **[2017]**

(A) channelizing oxygen to form ozone
(B) removal of oxygen by metabolic activity
(C) utilizing oxygen for membrane remodelling
(D) utilizing oxygen for synthesis of pentapeptide chain in peptidoglycan

**Sol.** **[B]**
Excess $O_2$ is used for metabolic activity.

**77.** Frederick Griffith performed an experiment where mice were killed when injected with a mixture of killed S-type *Streptococcus* (HKS) and live R-type *Streptococcus* (LRS) but not with HKS or LRS separately. Mice were killed because **[2017]**

(A) lipids from HKS made LRS virulent
(B) RNA from HKS transformed LRS and made it virulent
(C) proteins from HKS made LRS virulent
(D) DNA from HKS transformed LRS and made it virulent

**Sol.** **[D]**
Transformation occurs when DNA is taken up by R-strain form dead S-strain.

**78.** In diabetic patients, the pH of blood plasma can decrease leading to acidosis. This is because tissues catabolise **[2017]**

(A) amino acids leading to loss of buffering capacity of the blood
(B) stored glycogen leading to the accumulation of pyruvic acid
(C) stored fatty acids leading to the accumulation of beta hydroxybutyric acid and acetoacetic acid
(D) nucleic acid pool leading to decrease in blood pH

**Sol.** **[C]**

Lack of blood gulcose CEAO to break down of Fat which produce acetoacetic Acid and β-hydroxy butyric acid which decrease pH of blood.

**79.** If the number of alveoli in an individual is doubled without changing the total alveolar volume, the gas exchange capacity of the lungs will **[2017]**

(A) increase for both $O_2$ and $CO_2$

(B) decrease for both $O_2$ and $CO_2$

(C) remain unaltered for both $O_2$ and $CO_2$

(D) increase for $O_2$ and decrease for $CO_2$

**Sol.** **[A]**

Surface area will increase

**80.** In an experiment, bacteria were infected with $^{32}P$ labelled virus in a ratio of 5 : 1. The culture was rigorously shaken followed by centrifugation. Radioactivity was **[2017]**

(A) lost due to metabolic activity

(B) detected in supernatant as inorganic phosphate

(C) detected in the supernatant in association with viral capsid

(D) detected in bacterial cell pellet

**Sol.** **[D]**

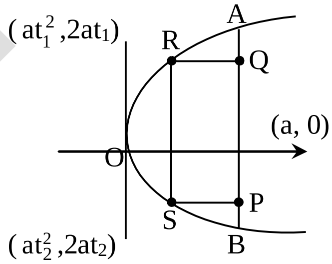
Bacteriophage infected bacteria found at the bottom containing viral DNA i.e. radioactive.

## Section 5 part B Mathematics

**81.** Let AB be the latus rectum of the parabola $y^2 = 4ax$ in the xy-plane. Let T be the region bounded by the finite arc AB of the parabola and the line segment AB. A rectangle PQRS of maximum possible area is inscribed in T with P, Q on line AB, and R, S on arc AB. Then area(PQRS)/area(T) equals

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$

**Sol.** **[D]**




$T = \int_0^a 2\sqrt{a}\ \ \sqrt{x}\ .\ dx = \frac{8a^2}{3}$

$t_1 = t_2$

Area of PQRS

$= 2a\ |t_1 - t_2| \times |a - at_1^2|$

But $t_1 = -t_2$

$= 4a^2 t_1(1 - t_1^2)$

Differentiation with respect $t_1$

We will get $t_1^2 = \frac{1}{3}$

Now put $t_1 = \frac{1}{\sqrt{3}}$ get Area of

PQRS = $\frac{8a^2}{3\sqrt{3}}$

ratio Becomes $\frac{1}{\sqrt{3}}$

**82.** Let A be the set of all permutations $a_1, a_2, \ldots, a_6$ of 1, 2, …., 6 such that $a_1, a_2, \ldots. a_k$ is not a permutation of 1, 2, …., k for any k, $1 \le k \le 5$. Then the number of elements in A is

(A) 192 (B) 408

(C) 312 (D) 528

**Sol. [D]**

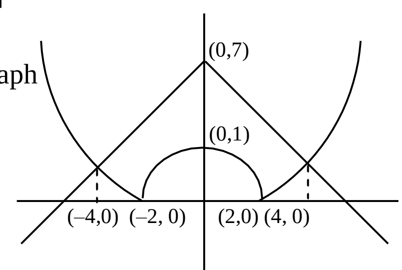
**83.** The area bounded by the curve $y = \frac{1}{4}|4 - x^2|$ and $y = 7 - |x|$ is

(A) 18 (B) 32

(C) 36 (D) 64

**Sol.** **[B]**

graph




Required Area

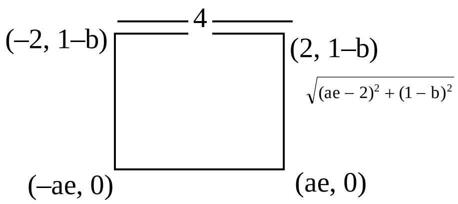
$$A = 2\left[\int_0^2 \left((7-x) - \frac{1}{4}(4-x^2)\right)dx + \int_2^4 \left((7-x) - \frac{1}{4}(x^2-4)dx\right)\right]$$

solve this = 32

**84.** An ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, a > b and the parabola $x^2 = 4(y + b)$ are such that the two foci of the ellipse and the end points of the latus rectum of parabola are the vertices of a square. The eccentricity of the ellipse is

(A) $\dfrac{1}{\sqrt{13}}$ (B) $\dfrac{2}{\sqrt{13}}$ (C) $\dfrac{1}{\sqrt{11}}$ (D) $\dfrac{2}{\sqrt{11}}$

**Sol.** **[B]**




As it is square

ae = 2

$4 = \sqrt{(ae-2)^2+(1-b)^2}$

b = –3 | b = 5

$a^2 = 13$ | $a^2 = 29$

(from realation $b^2 = a^2(1-e^2)$)

ae = 2

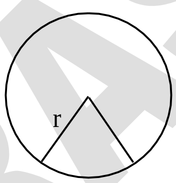
$e = \dfrac{2}{\sqrt{13}}$

**85.** A sector is removed from a metallic disc and the remaining region is bent into the shape of a circular conical funnel with volume $2\sqrt{3}\,\pi$.

The least possible diameter of the disc is

(A) 4 (B) 6 (C) 8 (D) 12

**Sol.** **[B]**




for a cone

r will be cone slant height

$V = 2\sqrt{3}$

Let x radius of cone

h be height then

$\dfrac{1}{3}\pi r^2 h = 2\sqrt{3}\,\pi$

$x^2h = 6\sqrt{3} \Rightarrow x^2 = \dfrac{6\sqrt{3}}{h}$

least Diameter $\Rightarrow$ least slant height of cone

$\ell^2 = x^2 + h^2$

$r^2 = x^2 + h^2$

$r^2 = \dfrac{6\sqrt{3}}{h} + h^2$

Diff. w. to x

$2r\dfrac{dr}{dh} = \dfrac{-6\sqrt{3}}{h^2} + 2h$

for maximum & minimum $\dfrac{dr}{dh} = 0$

$h = \sqrt{3} \Rightarrow h^2 = 3$

$x^2 = \dfrac{6\sqrt{3}}{\sqrt{3}}$

$x^2 = 6$

$r^2 = 6 + 3$

$r^2 = 9$

r = 3

d = 2r

d = 6

**86.** Let $g(x) = \displaystyle\int_0^{|x|^{2/4}} t^{2/3}\sin\frac{1}{t}\,dt$, for all real x.

Then $\displaystyle\lim_{x\to 0}\frac{g(x)}{x}$ is equal to

(A) $\infty$ (B) $-\infty$

(C) 0 (D) $\dfrac{3}{4}$

**Sol.** **[C]**

Apply L hospital rule

$g'(x) = |x|^{1/2}\sin\left(\dfrac{1}{|x|^{3/4}}\right)$

$\displaystyle\lim_{x\to 0} g'(X)$

$\displaystyle\lim_{x\to 0}|x|^{1/2}\sin\left(\frac{1}{|x|^{3/4}}\right) = 0$

**87.** Let $a_n = \displaystyle\int_{-\pi}^{\pi}|x-1|\cos nx\,dx$ for all natural numbers n. Then the sequence $(a_n)_{n\geq 0}$ satisfies

(A) $\displaystyle\lim_{n\to\infty} a_n = \infty$

(B) $\displaystyle\lim_{n\to\infty} a_n = -\infty$

(C) $\displaystyle\lim_{n\to\infty} a_n$ exists and is positive

(D) $\displaystyle\lim_{n\to\infty} a_n = 0$

**Sol.** **[D]**

$$a_n = \underbrace{\int_{-\pi}^{1} -(x-1)\cos nx\,dx}_{\text{I}} + \underbrace{\int_{1}^{\pi}(x-1)\cos nx\,dx}_{\text{II}}$$

Solve I & II part by I LATE

$$a_n = 2\pi \sin\frac{n\pi}{n} + \frac{2}{n^2}\cos n\pi - \frac{2}{n^2}\cos n$$

$$\lim_{x\to\infty} a_n = 0$$

**88.** Let f(x) be a polynomial with integer coefficients satisfying f(1) = 5 and f(2) = 7. The smallest possible positive value of f(12) is

(A) 5 (B) 7

(C) 27 (D) 15

**Sol.** **[C]**

f(x) = ax + b

5 = a + b ...(i)

7 = 2 a + b........................(ii)

solve a = 2

b = 3

f(x) = 2x + 3

f(12) = 24 +3

f(12) = 27

**89.** Suppose four balls labelled 1, 2, 3, 4 are randomly placed in boxes $B_1, B_2, B_3, B_4$. The probability that exactly one box is empty is

(A) $\frac{8}{256}$ (B) $\frac{9}{16}$

(C) $\frac{27}{256}$ (D) $\frac{9}{64}$

**Sol.** **[B]**

required probability =

$$\frac{{}^4C_1 \times \frac{4!}{(1!)^2 \times 2!} \times \frac{1}{2!} \times 3!}{4^4} = \frac{9}{16}$$

**90.** Let $f(x) = \log(1 + x^2)$ and A be a constant such that $\frac{|f(x) - f(y)|}{|x - y|} \le A$ for all x, y real and $x \neq y$. Then the least possible value of A is

(A) equal to 1

(B) bigger than 1 but less than 2

(C) bigger than 0 but less than 1

(D) bigger than 2

**Sol.** **[A]**

$$f'(x) = \frac{2x}{1+x^2}$$

Range of $\frac{2x}{1+x^2} \in [-1, 1]$

$\frac{|f(x) - f(y)|}{|x - y|} \le A$ means

maximum value of $\frac{|f(x) - f(y)|}{|x - y|}$ is always less than or equal to A.

So, least value of A is 1

## Section 6 part B Physics

**91.** One mole of an ideal monatomic gas undergoes the following four reversible processes :

Step 1 – it is first compressed adiabatically from volume 8.0 $m^3$ to 1.0 $m^3$.

Step 2 – then expanded isothermally at temperature $T_1$ to volume 10.0 $m^3$.

Step 3 – then expanded adiabatically to volume 80.0 $m^3$.

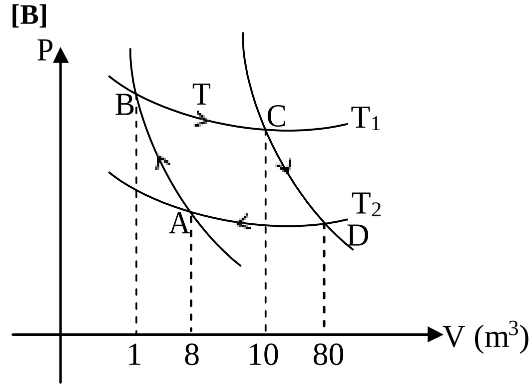
Step 4 – then compressed isothermally at temperature $T_2$ to volume 8.0 $m^3$.

Then $T_1/T_2$ is **[2017]**

(A) 2 (B) 4

(C) 6 (D) 8

**Sol.** **[B]**



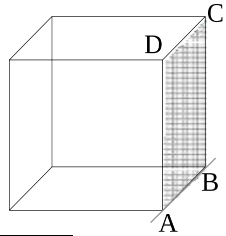

A to B

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{8}\right)^{\frac{5}{3}-1}$$

$$\frac{T_1}{T_2} = \left(\frac{8}{1}\right)^{\frac{2}{3}} = \frac{4}{1}$$

**92.** A solid cube of wood of side 2a and mass M is resting on a horizontal surface as shown in the figure. The cube is free to rotate about the fixed axis AB. A bullet of mass m (<<M) and speed v is shot horizontally at the face opposite to ABCD at a height 'h' above the surface to impart the cube an angular speed $\omega_c$ so that the cube just topples over. Then $\omega_c$ is (note : the moment of inertia of the cube about an axis perpendicular to the face and passing through the center of mass is $2Ma^3/3$) **[2017]**



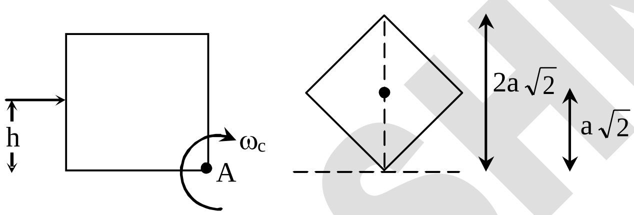

(A) $\sqrt{3gM / 2ma}$

(B) $\sqrt{3g / 4h}$

(C) $\sqrt{3g(\sqrt{2} - 1) / 2a}$

(D) $\sqrt{3g(\sqrt{2} - 1) / 4a}$

**Sol.** **[D]**




conservation of energy

$$\frac{1}{2} I_A \omega_c^2 = Mg\,(a\sqrt{2} - a).$$

$$I_A = I_{cm} + Ma^2 = \frac{2}{3}Ma^2 + M\,(a\sqrt{2})^2$$

$$I_A = \frac{8}{9}Ma^2$$

$$\frac{1}{2}\,\frac{8}{3}Ma^2\,\omega_c^2 = Mg\,a\,(\sqrt{2} - 1)$$

$$\omega_c = \sqrt{3g(\sqrt{2} - 1) / 4a}$$

**93.** A uniform thin wooden plank AB of length L and mass M is kept on a table with its B end slightly outside the edge of the table. When an impulse J is given to the end B, the plank moves up with centre of mass rising a distance 'h' from the surface of the table. Then- **[2017]**

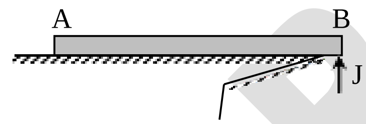
(A) $h > 9J^2/8M^2g$

(B) $h = J^2/2M^2g$

(C) $J^2/2M^2g < h < 9J^2/8M^2g$

(D) $h < J^2/2M^2g$

**Sol.** **[C]**




**Case-I**

Considering angular momentum w.r.t. end A

$\omega$ = Angular velocity just after impulse then

$$(\omega)\left(\frac{ML^2}{3}\right) = J(L)$$

$$\omega = \frac{3J}{ML}$$

$\Rightarrow$ velocity of CM

$$V_{cm_1} = \frac{\omega L}{2} = \frac{3J}{2M} \quad ...(1)$$

**Case-II**

Apply conservation o moment, if

$V_{cm_2}$ = Velocity of CM just after impulse

then $M\,V_{cm_2} = J$

$$V_{cm_2} = \frac{J}{M} \quad ...(2)$$

comparing (1) & (2)

$$V_{cm_2} < V_{cm_1}$$

$\Rightarrow$ velocity of CM just after impulse would be between above two extreme values

with (1), $Mgh_{max.} < \frac{1}{2}(M)\left(\frac{3J}{2M}\right)^2$

$$\Rightarrow \quad h_{max} < \frac{9J^2}{8M^2g} \quad ...(3)$$

with (2), $Mgh_{max} > \frac{1}{2}M\left(\frac{J}{M}\right)^2$

$$h_{max} > \frac{J^2}{2M^2g} \quad ...(4)$$

Use (3) & (4)

Hence $\frac{J^2}{2M^2g} < h_{max} < \frac{9J^2}{8M^2g}$

**94.** A square-shaped wire loop of mass m, resistance R and side 'a' moving with speed $v_0$, parallel to the x-axis, enters a region of uniform magnetic field B, which is perpendicular to the plane of the loop. The speed of the loop changes with distance x (x < a) in the filed, as **[2017]**

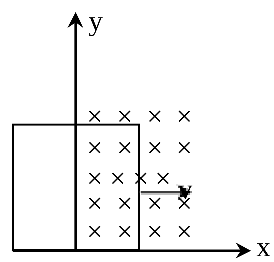
(A) $v_0 - \frac{B^2a^2}{Rm}x$

(B) $v_0 - \frac{B^2a^2}{2Rm}x$

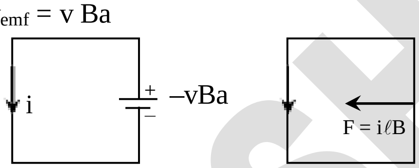
(C) $v_0 - \frac{B^2a}{Rm}x^2$

(D) $v_0$

**Sol.** **[A]**




$v_{emf} = v\,Ba$




$i = vBa/R$

$F = i\ell B = \frac{vBa \cdot Ba}{R}$

$ma = \frac{vB^2a^2}{R}$

$\frac{-mdv}{dt} = \frac{vB^2a^2}{R}$

$\Rightarrow -m\frac{vdv}{dx} = v\frac{B^2a^2}{R}$

$-m\int_{v_0}^{v} dv = \frac{B^2a^2}{R}\int_0^x dx$

$-m(v - v_0) = \frac{B^2a^2x}{R}$

$\Rightarrow v = v_0 - \frac{B^2a^2}{mR}x$

**95.** The emission series of hydrogen atom is given by **[2017]**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R is the Rydberg constant. For a transition from $n_2$ to $n_1$, the relative change $\Delta\lambda/\lambda$ in the emission wavelength if hydrogen is replaced by deuterium (assume that the mass of proton and neutron are the same and approximately 2000 times larger than that of electrons) is

(A) 0.025 % (B) 0.005 %

(C) 0.0025 % (D) 0.05 %

**Sol.** **[A]**

$R = \frac{m_e e^4}{8\varepsilon_0^2 ch^3}$

where $m_e$ = mass of electron

When we consider mass of nucleus also then we replace m with reduced mass

$\mu = \frac{m_e m}{m_e + m}$

where m = mass of nucleus

In case of hydrogen atom

$$\mu_1 = \frac{(m_e)(2000m_e)}{m_e + (2000)m_e} = \frac{2000}{2001}m_e$$

In case of deuterium

$$\mu_2 = \frac{(m_e)(4000m_e)}{m_e + (4000)m_e} = \frac{4000}{4001}m_e$$

Hence,

$$\frac{\lambda_2}{\lambda_1} = \frac{R_2}{R_1} = \frac{\mu_2}{\mu_1}$$

$$= \left(\frac{4000m_e}{4001}\right)\frac{(2001)}{(2000m_e)}$$

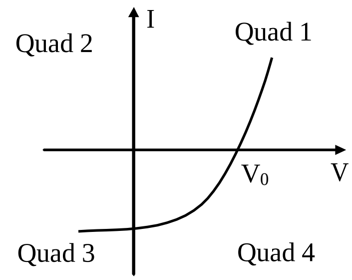
$$= \frac{4002}{4001}$$

$$\Rightarrow \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{1}{4001}$$

$$\therefore \frac{\Delta\lambda}{\lambda_1} \times 100 = \frac{1}{4001} \times 100$$

$$= 0.025\%$$

**96.** When light shines on a p-n junction diode, the current (I) vs, voltage (V) is observed as in the figure below : **[2017]**




In which quadrant(s) does the diode generate power, so that it can be used as a solar cell ?

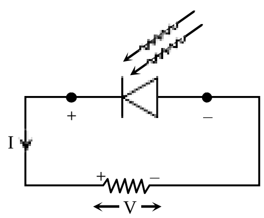
(A) Quad 1 only

(B) Quad 1 and 3 only

(C) Quad 4 only

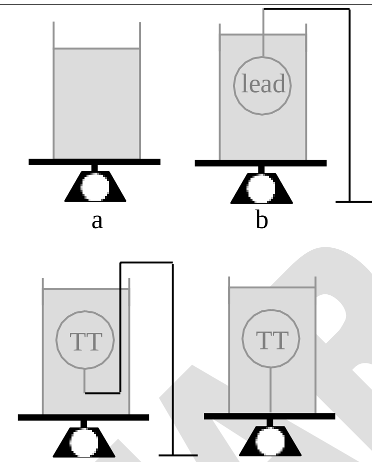
(D) Quad 1 and 4 only

**Sol.** **[C]**




both i and V are of same sign

**97.** Four identical beakers contain same amount of water as shown below. Beaker 'a' contains only water. A lead ball is held submerged in the beaker 'b' by string from above. A same sized plastic ball, say a table tennis (TT) ball, is held submerged in beaker 'c' by a string attached to a stand from outside. Beaker 'd' contains same sized TT ball which is held submerged from a string attached to the bottom of the beaker. These beakers (without stand) are placed on weighing pans and register reading $W_a$, $W_b$, $W_c$ and $W_d$ for a, b, c and d, respectively. (Effects of the mass and volume of the stand and string are to be neglected) **[2017]**




(A) $W_a = W_b = W_c = W_d$

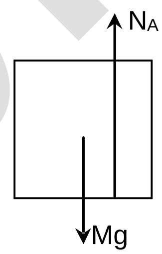
(B) $W_b = W_c > W_d > W_a$

(C) $W_b = W_c > W_a > W_d$

(D) $W_b > W_c > W_d > W_a$

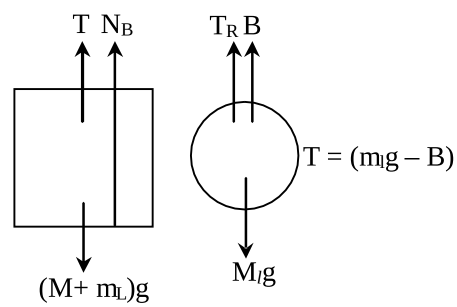
**Sol.** **[B]**

(A)




$N_A = Mg$

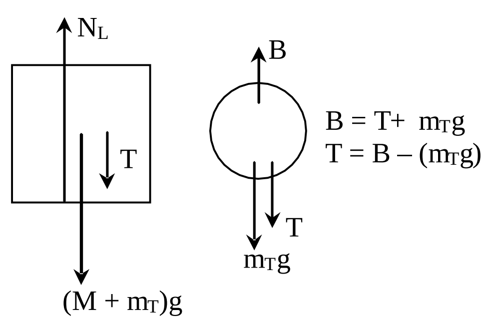
(B)




$(N_B = (M + M_L)g - T$

$N_B = Mg + B$

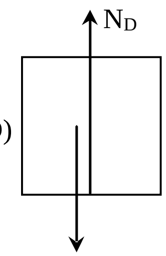
(C)



$N_C = T + (M + m_T)\, g$

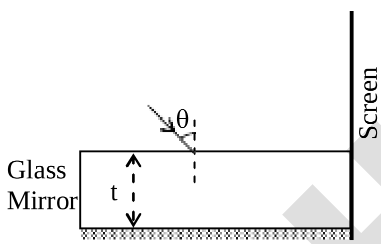
Therefore $N_L = B - m_T g + (M + m_T)g$

$N_L = B + Mg$

(D) 


$N_D = (M + m_T)\, g.$

**98.** Back surface of a glass (refractive index n and thickness t) is polished to work as a mirror as shown below. A laser beam falls on it and is partially reflected and refracted at the air-glass interface and fully reflected at the mirror surface respectively. A pattern of discrete spots of light is observed on the screen. **[2017]**

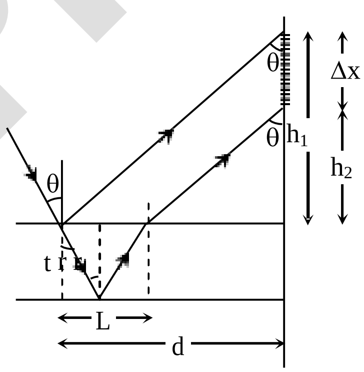



The spacing between the spots on the screen will be

(A) $\dfrac{2t\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$ (B) $\dfrac{2t\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$

(C) $\dfrac{2t\tan\theta}{\sqrt{n^2 - \sin^2\theta}}$ (D) $\dfrac{2t\sin\theta}{\sqrt{1 - \dfrac{\sin^2\theta}{n^2}}}$

**Sol.** **[A]**




Therefore $\Delta x = h_1 - h_2$

$\tan\theta = \dfrac{d}{h_1}$ & $\tan\theta = \dfrac{d - L}{h_2}$

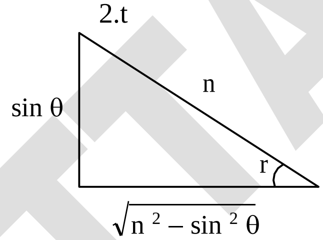
$\Delta x = \dfrac{d}{\tan\theta} - \dfrac{d - L}{\tan\theta}$

$\Delta x = \dfrac{L}{\tan\theta}$

Snell's Law

$1 \cdot \sin\theta = n \cdot \sin r \Rightarrow 1 \sin r = \dfrac{\sin\theta}{n}$

$\tan r = \dfrac{L}{2.t} \Rightarrow L = 2\, t \tan r$




$L = \dfrac{2t \cdot \sin\theta}{\sqrt{n^2 - \sin^2\theta}}$

$\therefore\ \Delta x = \dfrac{2t\sin\theta}{\sqrt{n^2 - \sin^2\theta}\,\tan\theta}$

$\tan\theta = \dfrac{2t\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$

**99.** Consider the following statements regarding the photoelectric effect experiment :

(I) Photoelectrons are emitted as soon as the metal is exposed to light

(II) There is a minimum frequency below which no photo-current is observed

(III) The stopping potential is proportional to the frequency of light

(IV) The photo-current varies linearly with the intensity of the light

Which of the above statements indicate that light consists of quanta (photons) with energy proportional to frequency ? **[2017]**

(A) I and III only

(B) II and III only

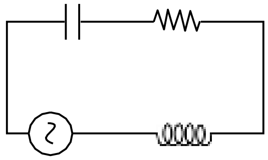
(C) II, III and IV only

(D) I, II and III only

**Sol.** **[D]**

Statement I, II & III are correct

**100.** Consider the R-L-C circuit given below. The circuit is driven by a 50 Hz AC source with peak voltage 220 V. If R = 400 Ω, C = 200 μF and L = 6 H, the maximum current in the circuit is closest to **[2017]**



(A) 0.120 A (B) 0.55 A
(C) 1.2 A (D) 5.5 A

**Sol.** **[A]**

$X_L = \omega L = 2\pi \times 50 \times 6 = 600\ \pi\Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi 80 \times 200 \times 10^{-6}} = \dfrac{100}{2\pi} = \dfrac{50}{\pi}\ \Omega$

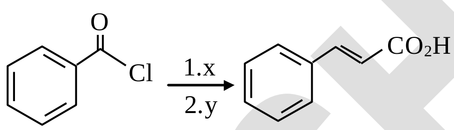
$I_{max} = \dfrac{V}{z} = \dfrac{220}{\sqrt{R^2 + (X_L - X_C)^2}}$

$= \dfrac{220}{\sqrt{400^2 + \left(600\pi - \dfrac{50}{\pi}\right)^2}}$

$I_{max} = 0.120$ A

## Section 7 Part B-Chemistry

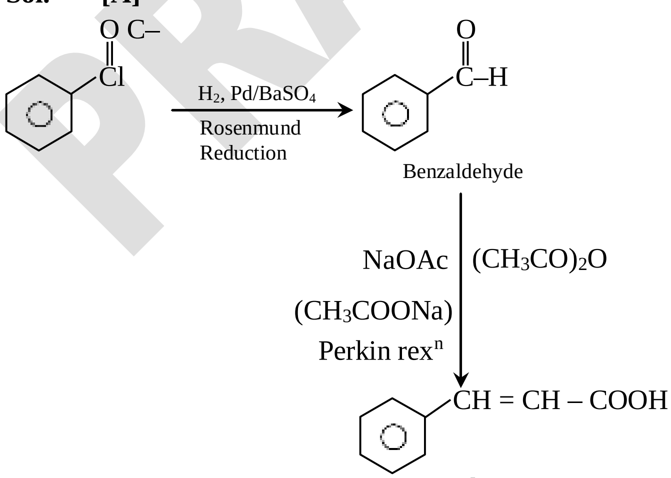
**101.** In the reaction



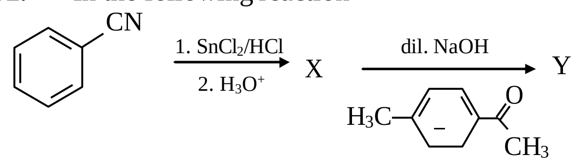

x and y are **[2017]**

(A) x = $H_2$, Pd/$BaSO_4$ ; y = NaOAc, $Ac_2O$
(B) x = $LiAlH_4$ ; y = NaOAc, $Ac_2O$
(C) x = $H_2$, Pd/C ; y = NaOH, $Ac_2O$
(D) x = $LiAlH_4$ ; y = NaOH, $Ac_2O$

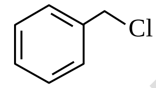
**Sol.** **[A]**

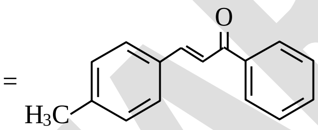


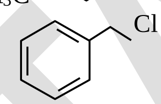

**102.** In the following reaction

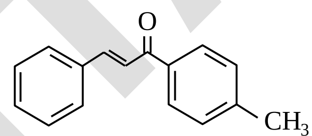


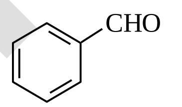

X and Y are **[2017]**

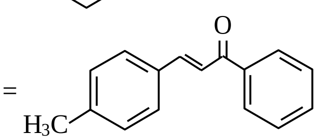
(A) X = 

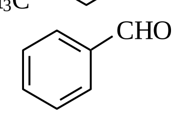
Y = 

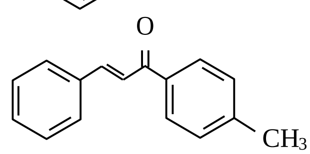
(B) X = 

Y = 

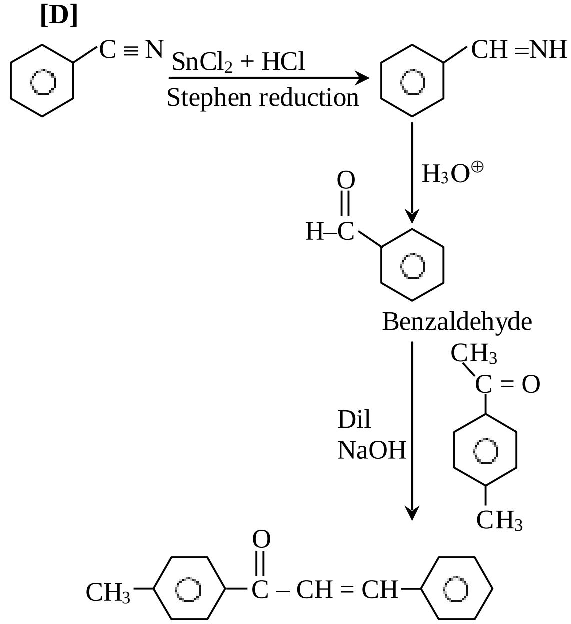
(C) X = 

Y = 

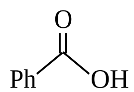
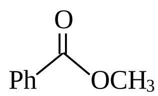
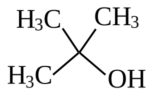
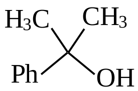
(D) X = 

Y = 

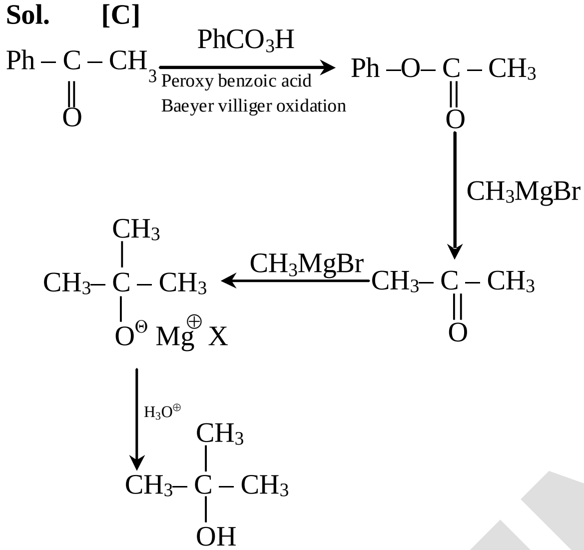
**Sol.** **[D]**



**103.** Acetophenone ($PhCOCH_3$) reacts with perbenzoic acid to produce a compound X. Reaction of X with excess $CH_3MgBr$ followed by treatment with aqueous acid predominantly produces **[2017]**

(A) 
(B) 
(C) 
(D) 

**Sol.** **[C]**




**104.** The fusion of chromite ore ($FeCr_2O_4$) with $Na_2CO_3$ in air gives a yellow solution upon addition of water. Subsequent treatment with $H_2SO_4$ produces an orange solution. The yellow and orange colours, respectively, are due to the formation of **[2017]**

(A) $Na_2CrO_4$ and $Na_2Cr_2O_7$
(B) $Cr(OH)_3$ and $Na_2Cr_2O_7$
(C) $Cr_2(CO_3)_3$ and $Fe_2(SO_4)_3$
(D) $Cr(OH)_3$ and $Na_2CrO_4$

**Sol.** **[A]**

$$8Na_2CO_3 + 4FeCr_2O_4 + 7O_2 \longrightarrow \underset{\text{yellow colour}}{Na_2CrO_4} + 2Fe_2O_3 + 8\,O_2$$

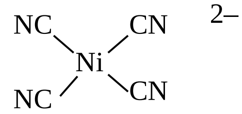
$$\underset{\text{yellow}}{2Na_2CrO_4} \xrightarrow{2H^+} \underset{\text{orange}}{Na_2Cr_2O_7} + 2Na^+ + H_2O$$

**105.** Hybridization and geometry of $[Ni(CN)_4]^{2-}$ are **[2017]**

(A) $sp^2d$ and tetrahedral
(B) $sd^3$ and square planar
(C) $sp^3$ and tetrahedral
(D) $dsp^2$ and square planar

**Sol.** **[D]**

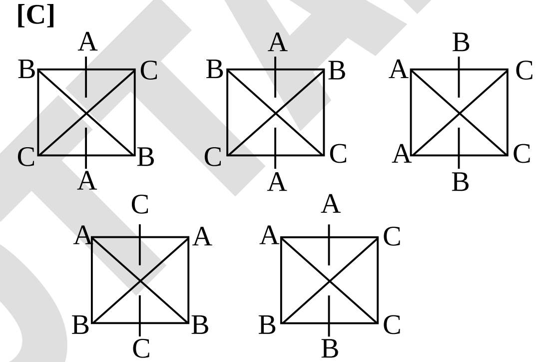
Hybridisation is $dsp^2$ and shape is square planar




**106.** The total number of geometrical isomers possible for an octahedral complex of the type $[MA_2B_2C_2]$ is
(M = transition metal ; A, B and C are monodentate ligands) **[2017]**

(A) 3 (B) 4
(C) 5 (D) 6

**Sol.** **[C]**



**107.** The maximum work (in kJ $mol^{-1}$) that can be derived from complete combustion of 1 mol of CO at 298 K and 1 atm is
[Standard enthalpy of combustion of CO = –283.0 kJ $mol^{-1}$ ; standard molar entropies at 298 K ; $S_{O2}$ = 205.1 J $mol^{-1}$, $S_{CO}$ = 197.7 J $mol^{-1}$, $S_{CO2}$ = 213.7 J $mol^{-1}$] **[2017]**

(A) 257 (B) 227
(C) 57 (D) 127

**Sol.** **[A]**

$$Co(g) + \frac{1}{2}O_2\,(g) \longrightarrow CO_2(g)\ \Delta N = -283.5$$

$$\Delta S = 213.7 - 197.7 - \frac{20501}{2} = -86.5$$

$$\Delta G = -283 - \frac{298\times(-86.5)}{1000} = -2570\text{ kJ}$$

$$W_{max} = -\Delta G = -(-257\text{ kJ}) = 257\text{ kJ}$$

**108.** 18 g of glucose ($C_6H_{12}O_6$) dissolved in 1 kg of water is heated to boiling. The boiling point (in K) measured at 1 atm pressure is closest to [Ebulioscopic constant, $K_b$ for water is 0.52 K kg $mol^{-1}$. Consider absolute zero to be –273.15ºC] **[2017]**

(A) 373.15 (B) 373.10
(C) 373.20 (D) 373.25

**Sol.** **[C]**

$\Delta T_b = K_b \times m$

$= 0.52 \times \dfrac{18/180}{1}$

$= 0.052$

$\therefore B \cdot P = 373.15 + 0.052$

$= 373.2\ K$

**109.** Polonium (atomic mass = 209) crystallizes in a simple cubic structure with a density of $9.32\ g\ cm^{-3}$. Its lattice parameter (in pm) is closest to **[2017]**

(A) 421 (B) 334

(C) 481 (D) 193

**Sol.** **[B]**

$d = \dfrac{N \times M}{N_A \times a^3}$

$9.32 = \dfrac{1 \times 209}{6.023 \times 10^{23} \times a^3}$

$\therefore a^3 = 37.2 \times 10^{-24S}$

$a = 3.33 \times 10^{-8}$ cm

$\cong 334$ pm

**110.** The following reaction takes place at 298 K in an electrochemical cell involving two metals A and B,

$A^{2+}(aq.) + B(s) \rightarrow B^{2+}(aq.) + A(s)$

With $[A^{2+}] = 4 \times 10^{-3}$ M and $[B^{2+}] = 2 \times 10^{-3}$M in the respective half-cells, the cell EMF is 1.091 V. The equilibrium constant of the reaction is closest to **[2017]**

(A) $4 \times 10^{36}$ (B) $2 \times 10^{37}$

(C) $2 \times 10^{34}$ (D) $4 \times 10^{37}$

**Sol.** **[B]**

$E_{cell} = E^\circ_{cell} - \dfrac{.0591}{2} \log \dfrac{2 \times 10^{-3}}{4 \times 10^{-3}}$

$1.091 = E^\circ_{cell} - \dfrac{.0591}{2} \log (.5)$

$E^\circ_{cell} = 1.099$

$E^\circ_{cell} = -\dfrac{.0591}{2} \log k$

$\log k = -\dfrac{1.099 \times 2}{.0591} = -37.22$

$\therefore k = 2 \times 10^{37}$

**Section 8 Part B Biology**

**111.** Suppose the three non-linked autosomal genes A, B and C control coat color in an animal and the dominants alleles A, B and C are responsible for dark color and the recessive alleles a, b and c are responsible for light color. If a cross between a male of AABBCC genotype and a female of aabbcc genotype produce 640 off springs in the $F_2$ generation, how many of them are likely to be of the parental genotype ? **[2017]**

(A) 10 (B) 20

(C) 160 (D) 640

**Sol. [B]**

$\dfrac{2}{64} \times 640 = 20$ (Trihybrid cross)

**112.** In a population of families having three children each, the percentage of population of families having both boys and girls is **[2017]**

(A) 10 (B) 25

(C) 50 (D) 75

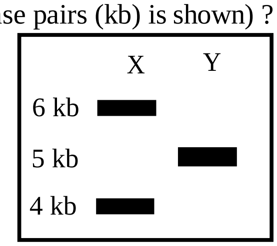
**Sol.** **[D]**

Probability of 2 Boy & 1 Girl = $\dfrac{3}{8}$

Probability of 1 Girl & 2 Boys = $\dfrac{3}{8}$

Total probability = $\dfrac{3}{8} + \dfrac{3}{8} = \dfrac{6}{8} = 75\%$

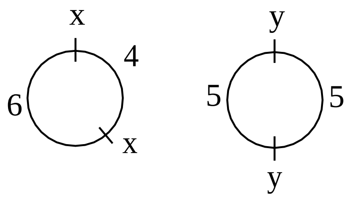
**113.** As indicated in the gel image, lanes X and Y represent samples obtained from a circular plasmid DNA after complete digestion using restriction enzyme X or Y with different sites, respectively. How many sites for X and Y are present in the plasmid (sizes of the bands in kilo base pairs (kb) is shown) ? **[2017]**




(A) 1 for X, 1 for Y (B) 2 for X, 1 for Y

(C) 1 for X, 2 for Y (D) 2 for X, 2 for Y

**Sol.** **[D]**




**114.** Matthew Meselson and Franklin Stahl grew *E.coli* (doubling time is 20 min) in medium containing $^{15}NH_4Cl$ for many generations. Then the *E.coli* was transferred to medium containing $^{14}NH_4Cl$. After 40 minutes, the cells were harvested and DNA was extracted and subjected to cesium chloride density gradient centrifugation. The proportion of light and hybrid DNA densities will be **[2017]**

(A) 50% light and 50% hybrid DNA
(B) 100% light DNA
(C) 100% hybrid DNA
(D) 25% light and 75% hybrid DNA

**Sol.** **[A]**
DNA replication is semiconservative

**115.** In a population interaction between the species X and the species Y, which ONE of the following statements is CORRECT ? **[2017]**

(A) When X benefits and Y is disadvantaged, it is Competition
(B) When both X and Y benefit, it is Mutualism
(C) When both X and Y are disadvantaged, it is Predation
(D) When both X and Y are disadvantaged, it is Parasitism

**Sol.** **[B]**
Both species are benefited mutualism

**116.** The protein P, the oligosaccharide O, and the oligonucleotide N are composed of 100 amino acid residues, 100 hexose residues, and 100 nucleotides, respectively. Which ONE of the following orders of molecular weights is CORRECT ? **[2017]**

(A) P > O > N (B) P > N > O
(C) N > O > P (D) O > P > N

**Sol.** **[C]**
Monomer of protein is amino acid oligosaccharide is sugar & oligonucleotide is nucleotides order of its molecular weight is Nucleotide > Monosaccharide > Amino acid

**117.** An octapeptide ($NH_2$-Asn-Glu-Tyr-Lys-Trp-Met-Glu-Gly) is subjected to complete protease and chemical digestion. Based on the results obtained, choose the INCORRECT option from below. **[2017]**

(A) Trypsin generates mixtures of dimer and trimer
(B) Trypsin generates tetramers only
(C) Cyanogen bromide generates a hexamer and a dimer
(D) Chymotrypsin generates mixture of dimer and trimers

**Sol. [A]**
**Cleavage site :-** for Trypsin-After Lys & Arg
For chymotrypsin - After Phe, Trp, or Tyr
For cyanogen bromide- After met

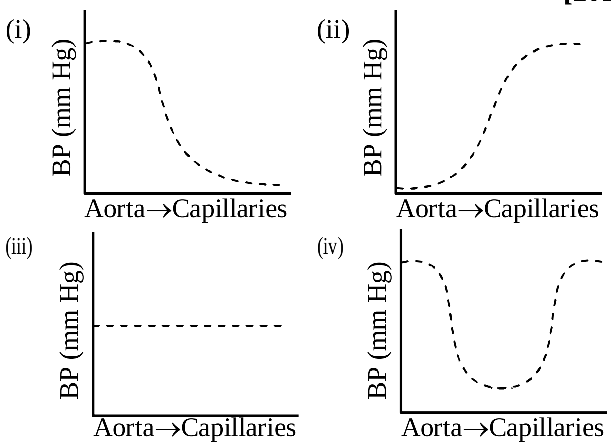
**118.** Match the enzymes in column-I with their respective biochemical reactions in column-II. Choose the CORRECT combination from below **[2017]**

| Column-I | Column-II |
|---|---|
| (P)Transaminases acid | (i) removal of phosphoryl group from a specific amino |
| (Q) Protein Kinases acid | (ii) removal of α-amino group from a specific amino |
| (R) Protein Phosphatases acid | (iii) addition of phosphoryl group to a specific amino |
| (S) Dehydrogenases | (iv) interconversion of optical isomers |
| | (v) oxidation and reduction of substrates |

(A) P-iv, Q-ii, R-iii, S-v
(B) P-ii, Q-i, R-ii, S-iv
(C) P-ii, Q-iii, R-i, S-v
(D) P-v, Q-ii, R-iii, S-i

**Sol.** **[C]**
Fact based

**119.** Which ONE of the following graphs best describes the blood pressure (BP) change when blood moves from aorta to capillaries ? **[2017]**

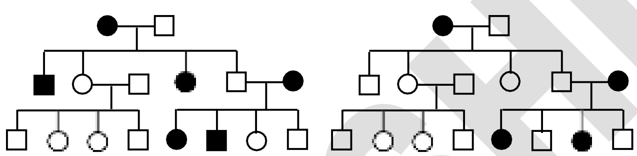



(A) (i) (B) (ii)
(C) (iii) (D) (iv)

**Sol.** **[A]**
Blood pressure decrease as it channelise in numerous fine blood vessel.

**120.** The following two pedigrees describe the autosomal genetic disorders P and Q in Family 1 and Family 2, respectively **[2017]**

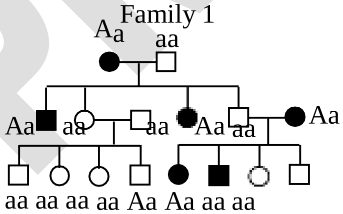
Family 1 Family 2



Choose the CORRECT statement from the following options.
(A) Both P and Q are dominant traits
(B) P is a dominant trait and Q is a recessive trait
(C) Both P and Q are recessive traits
(D) P is a recessive trait and Q is a dominant trait

**Sol.** **[B]**

Family 1




Family 2